DIGITAL TWINS OF CIVIL STRUCTURES USING NEURAL NETWORKS AND PROBABILISTIC GRAPHICAL MODELS

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This work proposes a predictive digital twin approach to the health monitoring and management planning of civil structures. The asset-twin dynamical system and its evolution over time are encoded by means of a probabilistic graphical model, adopted to rule the observations-to-decisions flow and quantify the related uncertainty. Deep learning models are adopted to assimilate observational data, and provide structural health diagnostics in real-time. The digital state is updated in a sequential Bayesian inference fashion, to inform an optimal planning of maintenance and management actions. A preliminary offline phase involves the population of training datasets through a reduced-order numerical model, and the computation of a health-dependent control policy.

Keywords: *digital twin, structural health monitoring, Bayesian network, deep learning, model order reduction*

1. Introduction

The digital twin (DT) concept represents the most exciting opportunity to move forward predictive maintenance practices, and thus increase the safety and availability of civil structures. This is nowadays possible as the installation of data collecting systems has become affordable, and thanks to the advances in learning methodologies.

This work proposes a DT framework for civil structures. The asset-twin dynamical system is encoded by means of a dynamic Bayesian network (DBN) inspired by [1]. The observations-to-decisions flow is encoded as:

- *From physical to digital.* Observational data, or measurements, are gathered from the physical system and assimilated with deep learning (DL) models, see e.g. [2], to estimate the structural health parameters underlying the digital state and describing the variability of the physical asset. This first estimate of the digital state is then exploited to estimate an updated digital state, according to control-dependent transition dynamics models describing how the structural health is expected to evolve.
- *From digital to physical.* The updated digital state is employed to predict the future evolution of the physical system, thereby enabling predictive decision making about maintenance and management actions.

The DT framework is made computationally efficient through a preliminary offline phase that involves: (i) the population of training datasets through a reduced-order numerical model, see e.g. [3], exploiting the physics-based knowledge about the system response. This is useful to overcome the lack of experimental data typical of civil engineering applications. (ii) training the DL models underlying the structural health identification. This allows for automating the selection and extraction of optimized damage-sensitive features, to ultimately relate them with the corresponding structural states in real-time. (iii) learning the health-dependent control policy to be applied at each time step of the online phase, to map the belief over the digital state onto actions feeding back to the physical asset.

The strategy is assessed on the simulated monitoring of a railway bridge, demonstrating the capabilities of health-aware DTs of accurately tracking the evolution of structural health parameters under varying operational conditions, and promptly suggesting the most appropriate control input with relatively low uncertainty.

2. Methodology

The DT assimilates vibration recordings shaped as multivariate time histories $\mathbf{U}(\boldsymbol{\mu}) = [\mathbf{u}_1, \dots, \mathbf{u}_{N_u}] \in \mathbb{R}^{L \times N_u}$, consisting of N_u time series made of L sensor measurements. To this aim, a simulation-based strategy is adopted to train DL models on vibration response data, exploiting physics-based models. The asset to be monitored is modeled as a linear-elastic continuum, discretized in space through finite elements. Its dynamic response to the applied loadings is described by the semi-discretized form of elasto-dynamics. The model is parametrized through a vector $\boldsymbol{\mu} \in \mathbb{R}^{N_{\text{par}}}$ of N_{par} parameters ruling the operational and damage conditions. Damage is modeled as a localized reduction of the material stiffness, obtained by means of two variables $y \in \mathbb{N}$ and $\delta \in \mathbb{R}$, respectively describing its position, among a set of predefined locations $y = 0, \dots, N_y$, and its magnitude. To speed-up the generation of synthetic datasets, a reduced-order model, relying on a reduced basis method, is adopted in place of the finite element model. The training dataset $\mathcal{D}$ is then populated with I instances as $\mathcal{D} = \{(\mathbf{U}_i, y_i, \delta_i)\}_{i=1}^{I}$.

In order to detect, locate, and quantify the presence of structural damage, a classification DL model $\text{NN}_{\text{CL}} : \mathbf{U} \to y$ is adopted to address damage detection/localization, and regression DL models $\text{NN}_{\text{RG}}^{j} : \mathbf{U} \to \delta$, with $j = 1, \dots, N_y$, are subsequently adopted to address damage quantification.

The DBN defining the asset-twin dynamical system is sketched in Fig. 1. The physical state $S_t \sim P(s_t)$, encapsulates the variability in the state of the asset. The digital state $D_t \sim P(d_t)$ is instead characterized by the structural health parameters adopted to capture such a variability. The observed data $O_t = o_t$ are assimilated with the DL models, to provide a first estimate of the digital state $D_t^{\text{NN}} \sim P(d_t^{\text{NN}})$. This is then adopted in a Bayesian inference fashion, to update the prior belief D_{t-1} and estimate an updated digital state D_t. This can thus be exploited to compute quantities of interest $Q_t \sim p(q_t)$ and to suggest the next control input. $U_t \sim P(u_t)$ and $U_t^A = u_t^A$ denote the belief about what action to take and the control input effectively enacted, respectively. U_t is estimated according to a control policy, that is computed offline by solving the planning problem induced by the expected evolution of the structural health. This involves maximizing the reward $R_t \sim p(r_t)$ quantifying the asset performance over the planning horizon. By exploiting the conditional independence resulting from the graph topology and the Bayes rule, the joint distribution over variables is factorized up to the current time step t_c, as:

$$\begin{aligned}&p(D_0^{\text{NN}}, \dots, D_{t_c}^{\text{NN}}, D_0, \dots, D_{t_c}, Q_0, \dots, Q_{t_c}, R_0, \dots, R_{t_c}, U_0, \dots, U_{t_c} | o_0, \dots, o_{t_c}, u_0^A, \dots, u_{t_c}^A) \\ &\propto \prod_{t=0}^{t_c} \left[\phi_t^{\text{data}} \phi_t^{\text{history}} \phi_t^{\text{NN}} \phi_t^{\text{QoI}} \phi_t^{\text{control}} \phi_t^{\text{reward}} \right],\end{aligned} \tag{1}$$

$$\begin{aligned}&\phi_t^{\text{data}} = P(O_t = o_t | D_t^{\text{NN}}), && \phi_t^{\text{history}} = P(D_t | D_{t-1}, U_{t-1}^A = u_{t-1}^A), && \phi_t^{\text{NN}} = P(D_t | D_t^{\text{NN}}), \\ &\phi_t^{\text{QoI}} = p(Q_t | D_t), && \phi_t^{\text{reward}} = P(R_t | D_t, U_t^A = u_t^A), && \phi_t^{\text{control}} = P(U_t | D_t).\end{aligned} \tag{2}$$

Since the spaces of the unobserved variables is discrete, we can propagate and update the relative belief exactly with a single pass of the sum-product algorithm. Starting from the updated digital state D_{t_c}, prediction in future is then achieved by unrolling until a desired prediction time the portion of the graph relative to D_t, Q_t, R_t and U_t.

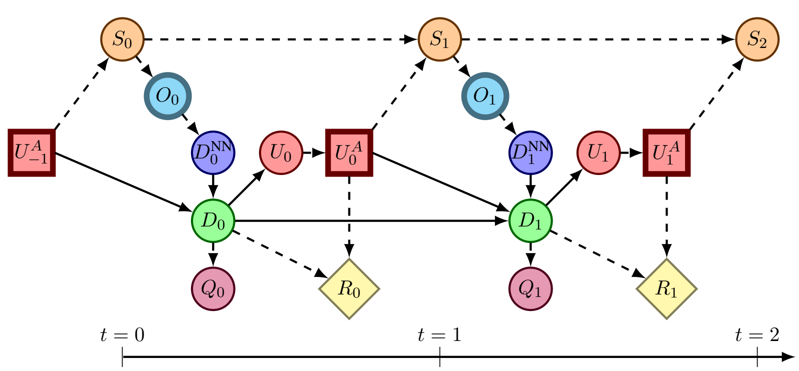
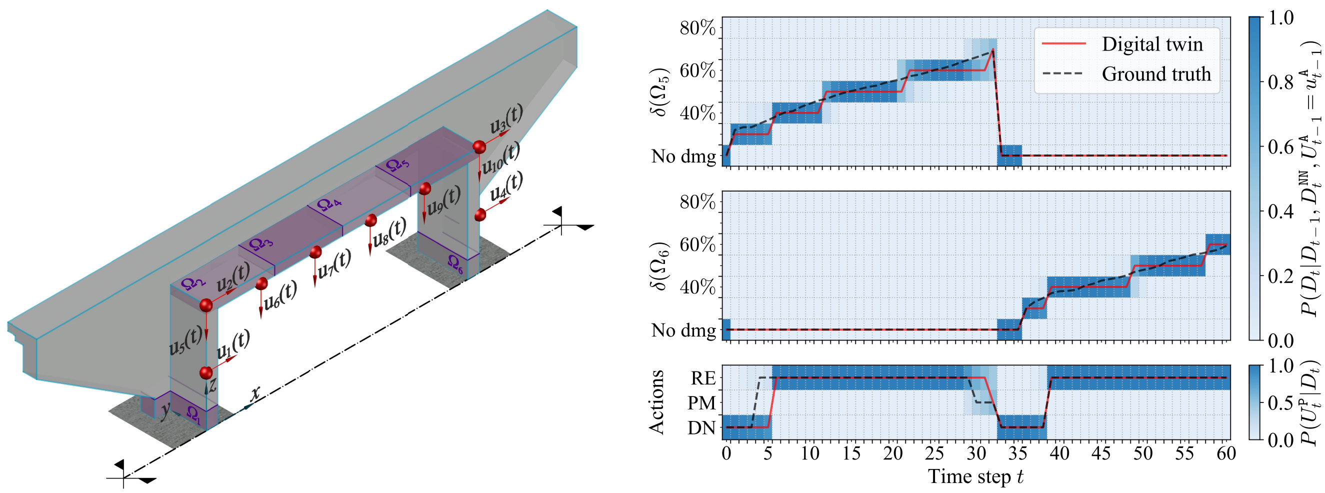


Figure 1: Adopted DBN: circle nodes denote random variables, square nodes actions, and diamond nodes the objective function. Bold outlines denote observed quantities, thin outlines estimated quantities. Solid edges represent dependencies encoded via conditional probability tables, dashed edges encode out-of-the-graph computations.

3. Simulated experiment

The DT framework is applied to the simulated monitoring of a railway bridge. Synthetic vibration recordings $\mathbf{U}$ are obtained from $N_u = 10$ sensors deployed as depicted in Fig. 2a. In addition to the undamaged condition, damage is accounted for by means of a stiffness reduction that can take place within $N_y = 6$ predefined subdomains Ω_j, $j = 1, \dots, N_y$, with magnitude $\delta \in [30\%, 80\%]$. The structural health parameters underlying the digital state are therefore $d = (y, \delta)$. We consider the following control inputs: do nothing (DN) – the physical state evolves according to a stochastic deterioration process; perfect maintenance (PM) – the asset returns to the damage-free state; restrict operational conditions (RE) – only light weight trains are allowed to travel across the bridge, yielding a lower deterioration rate and a lower revenue. Fig. 2b reports a sample DT simulation, in which the variation in the digital state is estimated every time a train travels across the bridge. Damage initially develops within Ω_5. The RE action is suggested as soon as the DT estimates a $\delta \geq 35\%$, after which point the DT keeps on tracking the structural health evolving with a lower deterioration rate. A PM action is finally suggested due to an excessively compromised structural state. A similar behavior can be observed for the following damage scenario affecting Ω_6.



(a) (b)

Figure 2: (a) details of synthetic recordings related to displacements $u_1(t), \dots, u_{10}(t)$, and predefined damageable regions $\Omega_1, \dots, \Omega_6$; (b) probabilistic and best point estimates of (top) digital state evolution against the ground truth digital state, and (bottom) informed control inputs against the optimal control input under ground truth.

4. Conclusions

In this work, we have proposed a predictive digital twin approach to the health monitoring of civil structures, to move forward predictive maintenance practices. The obtained results have demonstrated the digital twin capabilities of tracking the digital state with relatively low uncertainty, and promptly suggesting the appropriate control input.

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