

19-20 October, Rome, Italy

DIGITAL TWINS OF CIVIL STRUCTURES USING NEURAL NETWORKS AND PROBABILISTIC GRAPHICAL MODELS

Matteo Torzoni

Dipartimento di Ingegneria Civile e Ambientale
Politecnico di Milano

matteo.torzoni@polimi.it

Joint work with

Marco Tezzele, Stefano Mariani, Andrea Manzoni, Karen E. Willcox



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ODEN INSTITUTE

FOR COMPUTATIONAL ENGINEERING & SCIENCES

The need of Structural Health Monitoring (SHM)

Aloha Airlines 1988, separation of the fuselage upper lobe – 1 fatality



Genoa 2018, partial collapse of the Morandi Bridge – 43 fatalities



What is SHM?

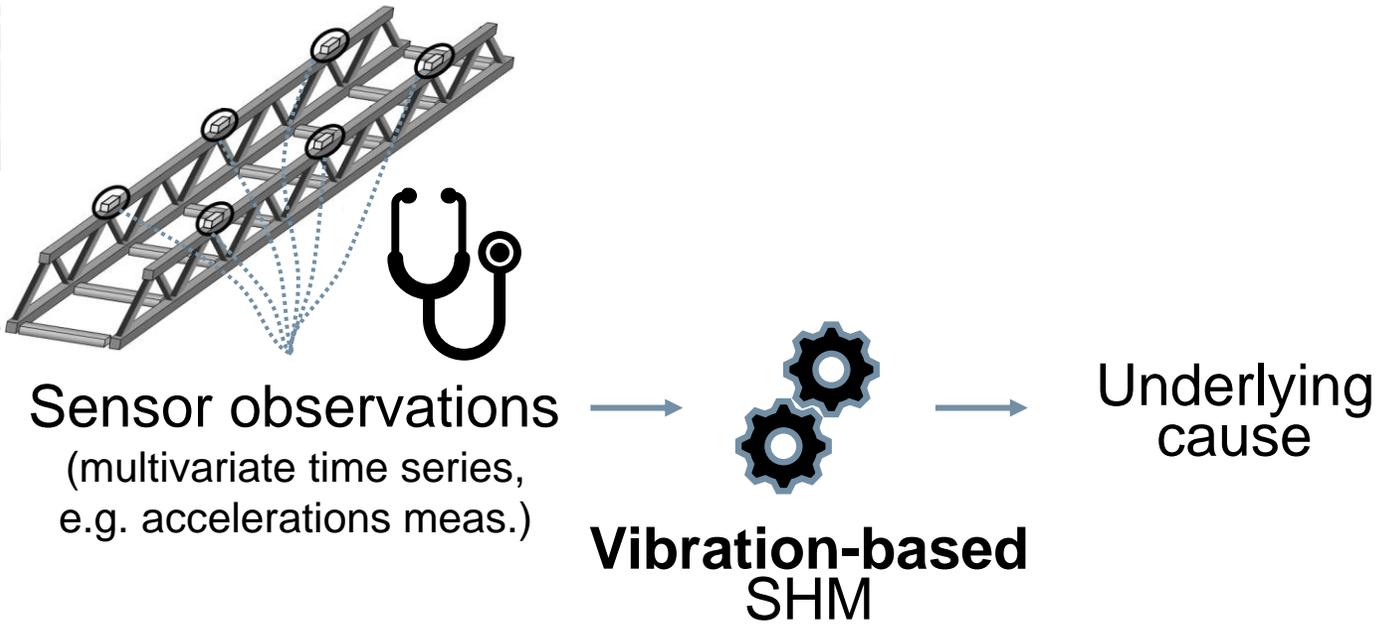
The implementation of a damage detection strategy for deteriorating structures.



Why SHM?

Optimal management: to reduce lifecycle costs and to increase the system safety and availability.

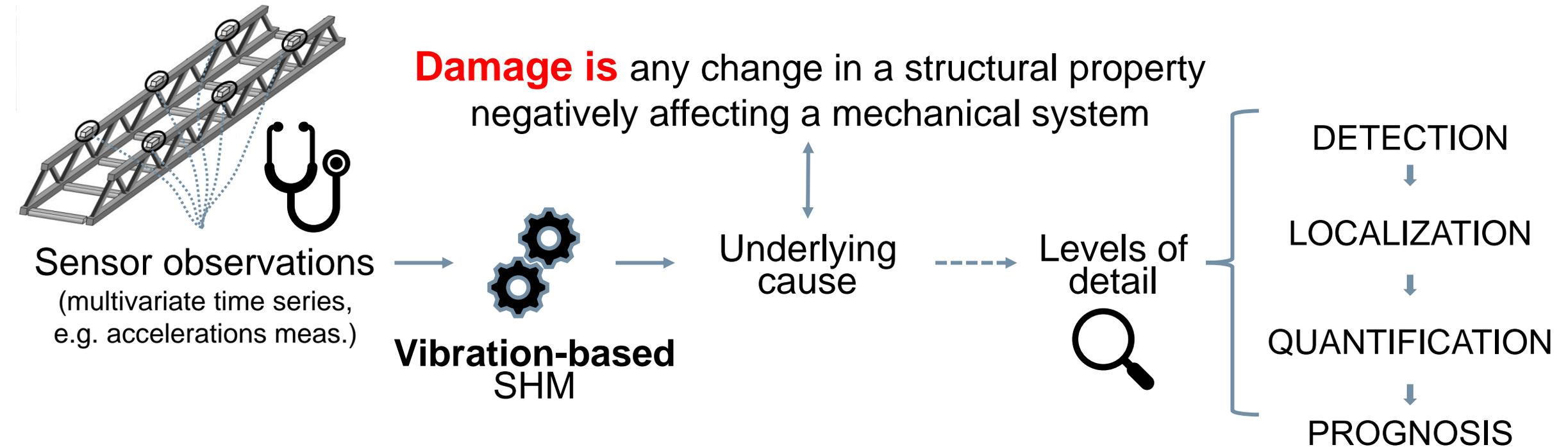
State-of-art: workflow & SHM hierarchical structure



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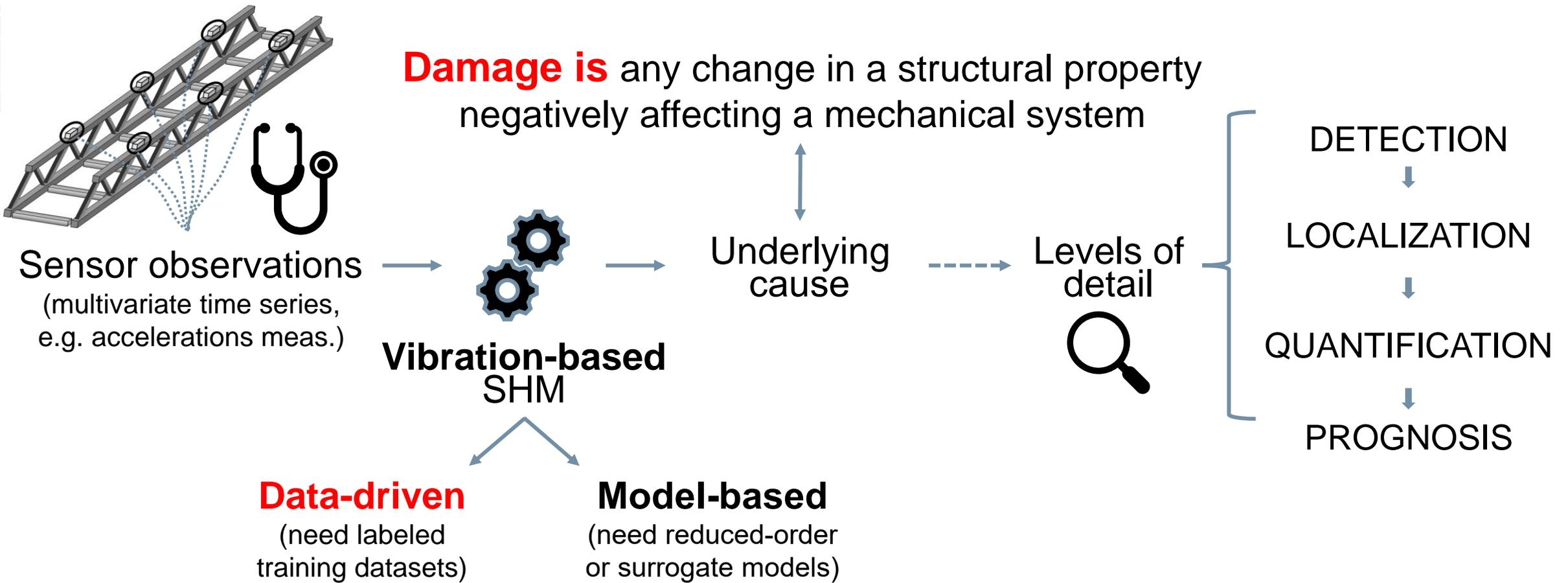
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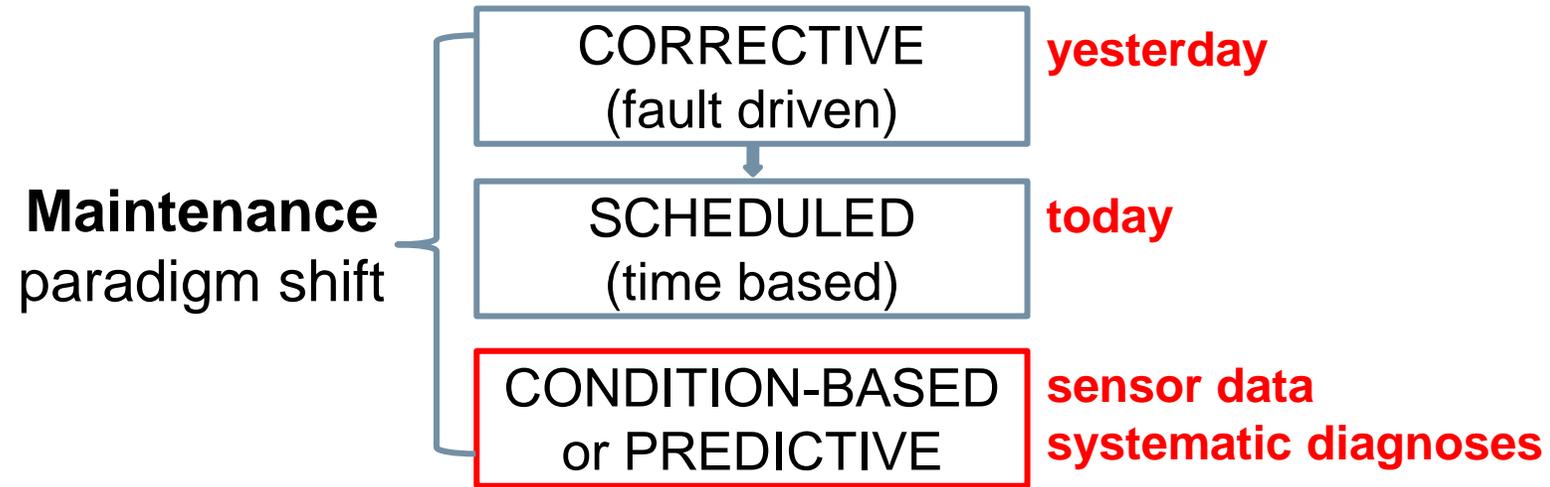
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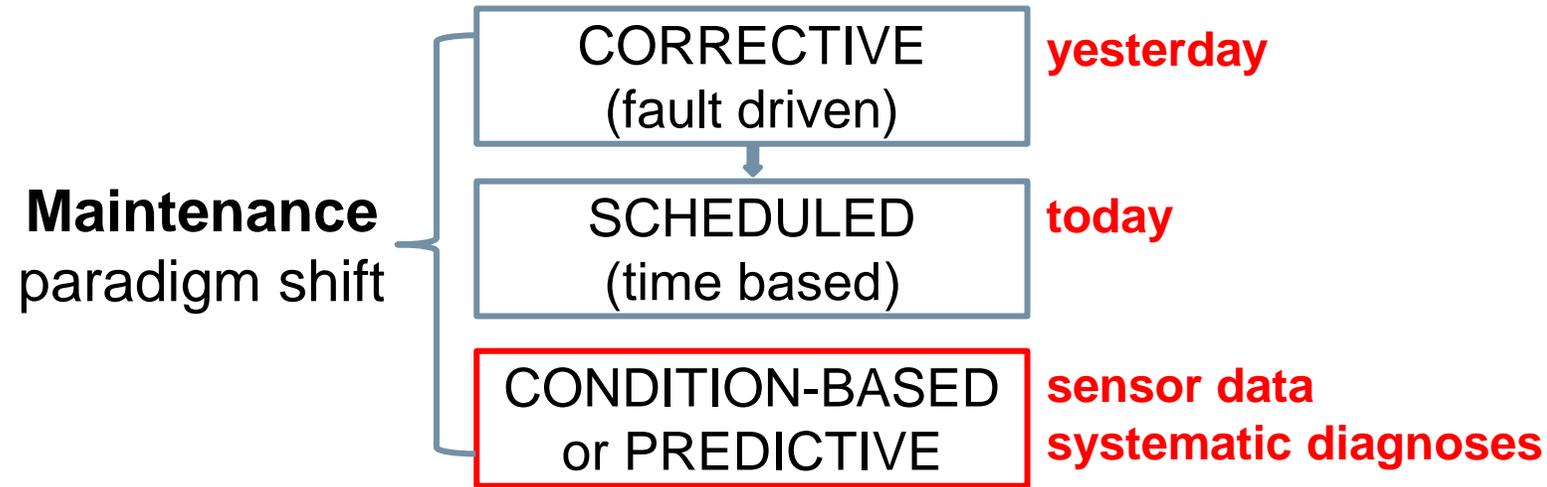


Which data? → Physics-based models (localization, quantification)

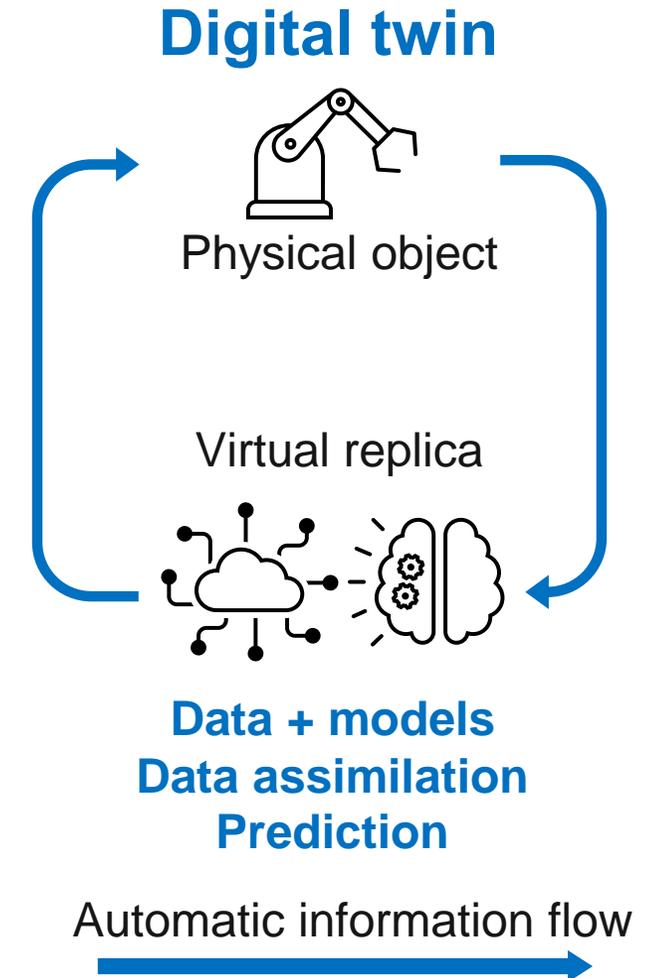
SHM for optimal management of deteriorating structures



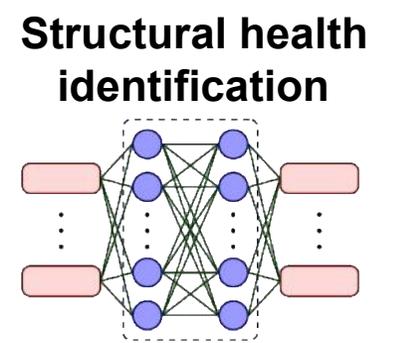
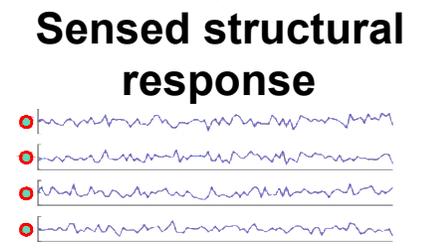
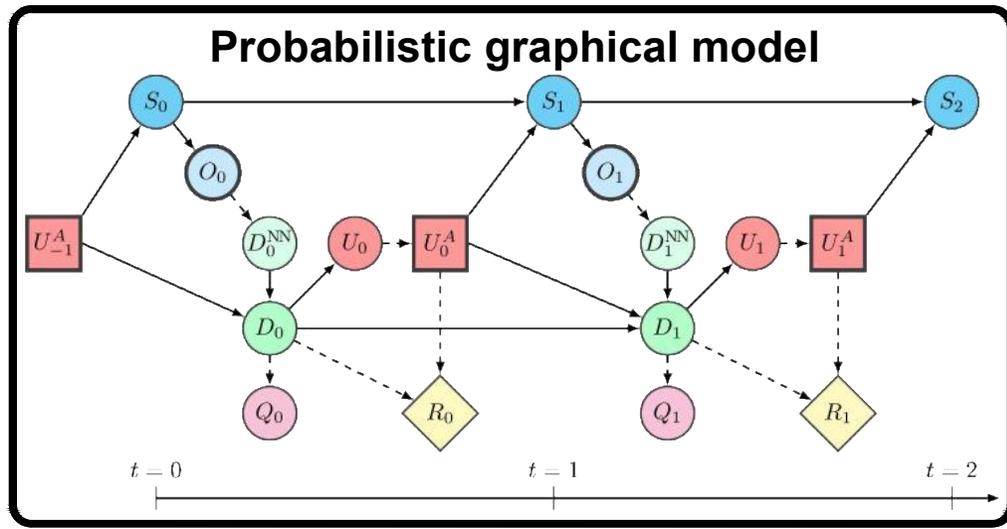
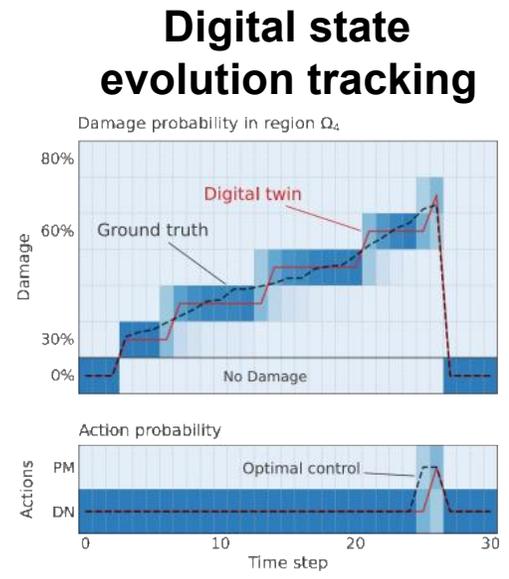
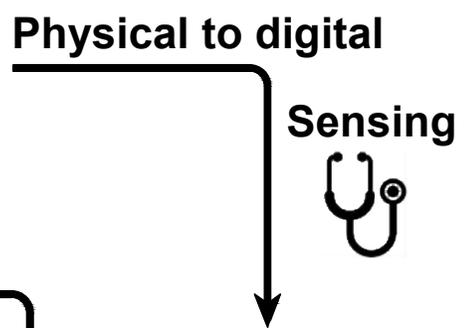
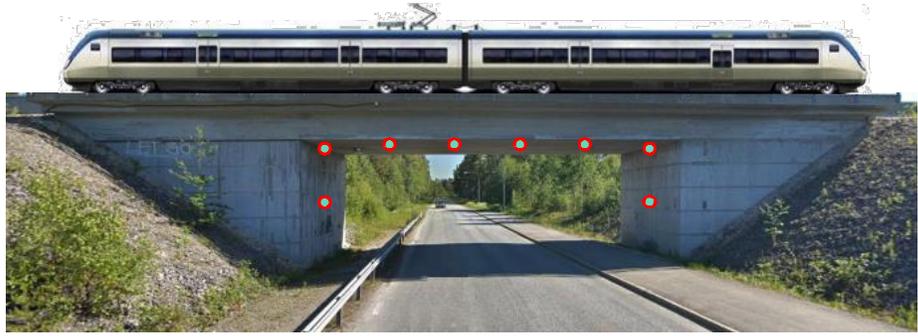
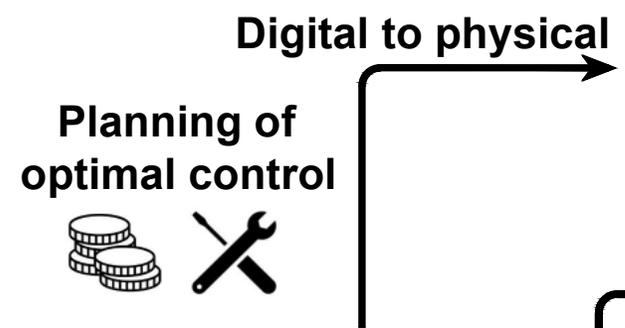
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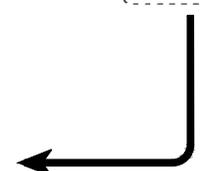
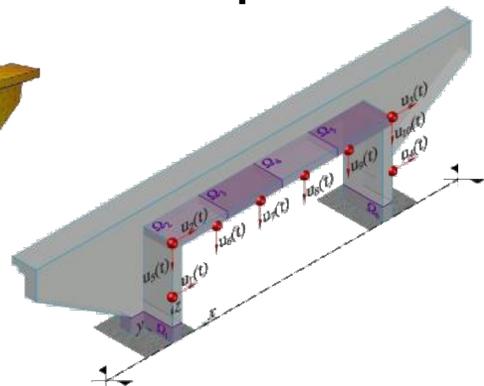
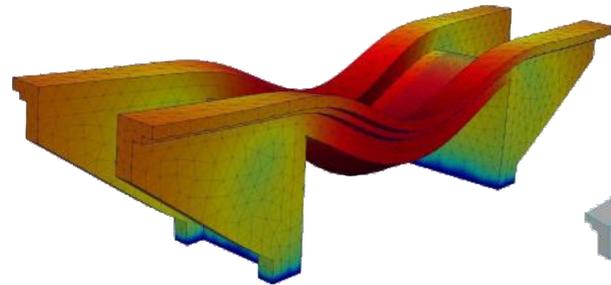
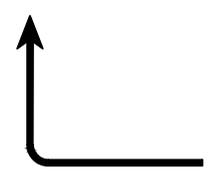
Goal: create a digital twin that adapts to the evolving structural health providing real-time health diagnostics that enable dynamic decision making about management and maintenance actions.



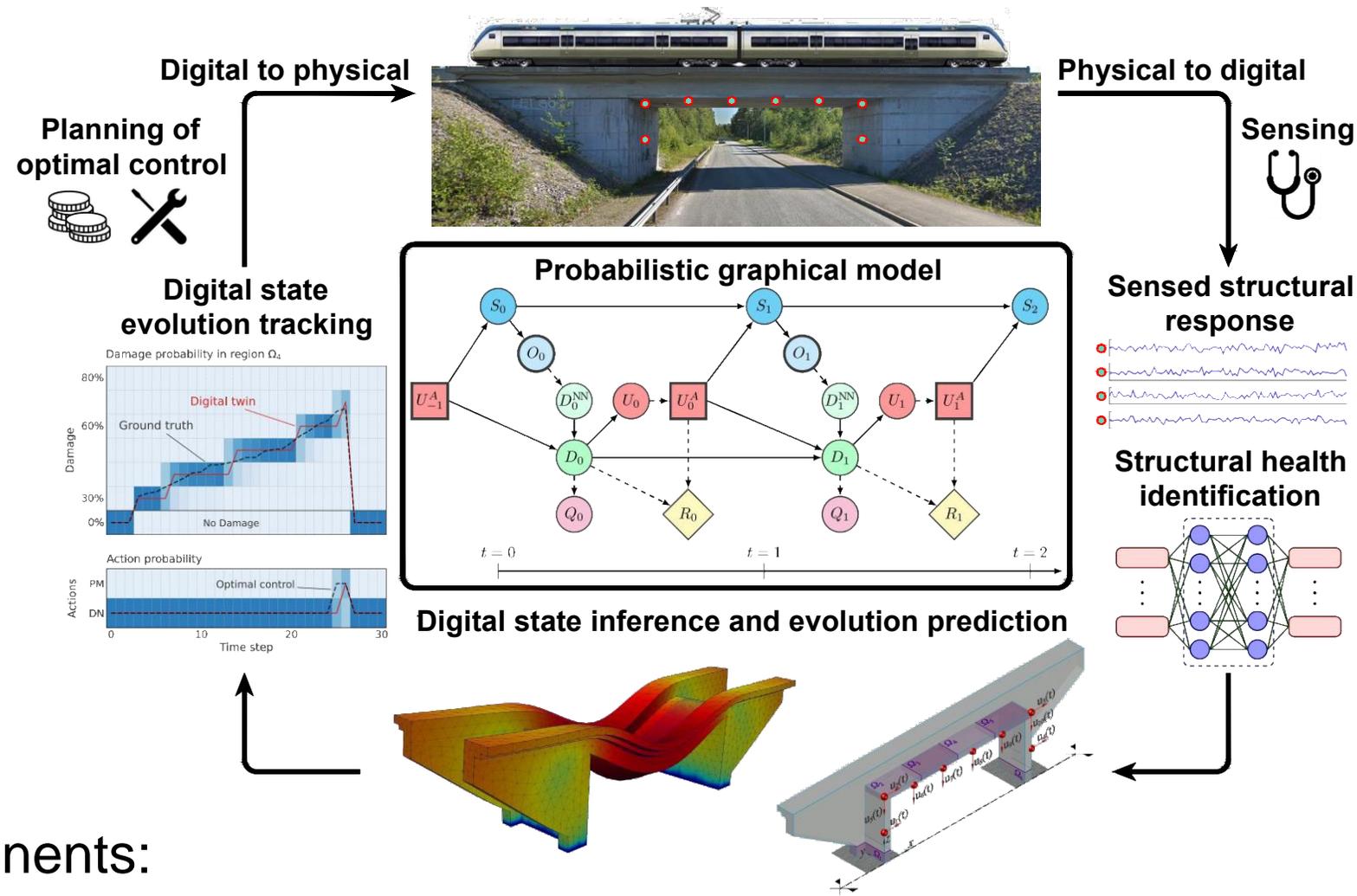
Overview: end-to-end information flow



Digital state inference and evolution prediction



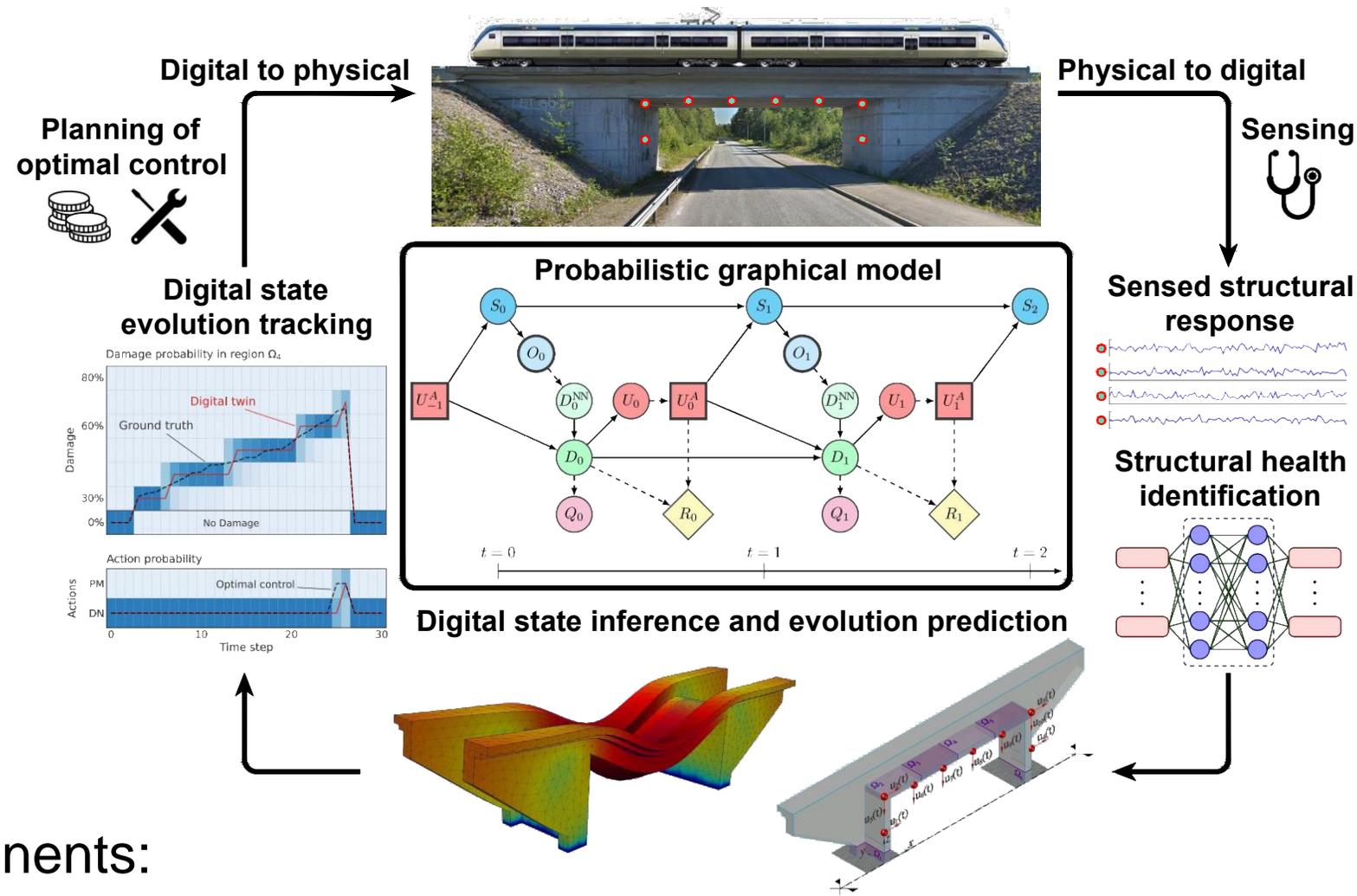
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Main components:

- ❖ Simulation-based damage identification
- ❖ Structural health identification using neural networks
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Physics-based models to simulate the effect of damage

Governing equation of motion

$$\begin{cases} \rho \ddot{\mathbf{v}} + \eta \dot{\mathbf{v}} - \nabla \cdot \boldsymbol{\sigma}(\mathbf{v}, \boldsymbol{\mu}) = \mathbf{b}(\mathbf{x}, t, \boldsymbol{\mu}) & \text{in } \Omega \times (0, T) \\ \mathbf{v} = \mathbf{g}_D(\mathbf{x}, t, \boldsymbol{\mu}) & \text{on } \Gamma_D \times (0, T) \\ \boldsymbol{\sigma}(\mathbf{v}, \boldsymbol{\mu}) \cdot \mathbf{n} = \mathbf{g}_N(\mathbf{x}, t, \boldsymbol{\mu}) & \text{on } \Gamma_N \times (0, T) \\ \mathbf{v}(t=0) = \mathbf{v}_0(\mathbf{x}) & \text{in } \Omega \\ \dot{\mathbf{v}}(t=0) = \dot{\mathbf{v}}_0(\mathbf{x}) & \text{in } \Omega \end{cases}$$

Linearized kinematics

$$\boldsymbol{\varepsilon}(\boldsymbol{\mu}) = \frac{1}{2} [\nabla \mathbf{v}(\boldsymbol{\mu}) + (\nabla \mathbf{v}(\boldsymbol{\mu}))^\top]$$

Linear-elastic material

$$\boldsymbol{\sigma}(\boldsymbol{\mu}) = \mathbf{D}(\boldsymbol{\mu}) \boldsymbol{\varepsilon}(\mathbf{v}(\boldsymbol{\mu}))$$

Physics-based model describing the dynamic response of a structure to the applied loadings

Finite element space discretization

$$\begin{cases} \mathbf{M} \ddot{\mathbf{x}}(t) + \mathbf{C}(\boldsymbol{\mu}) \dot{\mathbf{x}}(t) + \mathbf{K}(\boldsymbol{\mu}) \mathbf{x}(t) = \mathbf{f}(t, \boldsymbol{\mu}), & t \in (0, T) \\ \mathbf{x}(0) = \mathbf{x}_0 \\ \dot{\mathbf{x}}(0) = \dot{\mathbf{x}}_0 \end{cases}$$

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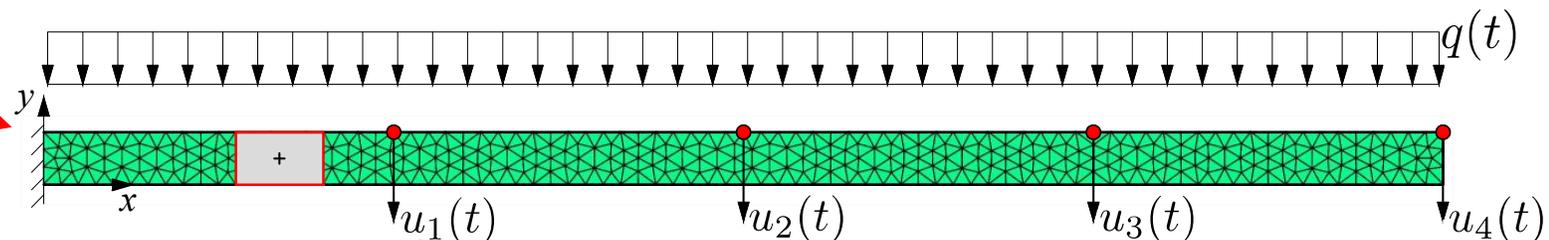
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Parameter vector $\boldsymbol{\mu}$: damage, loadings, environment, ...



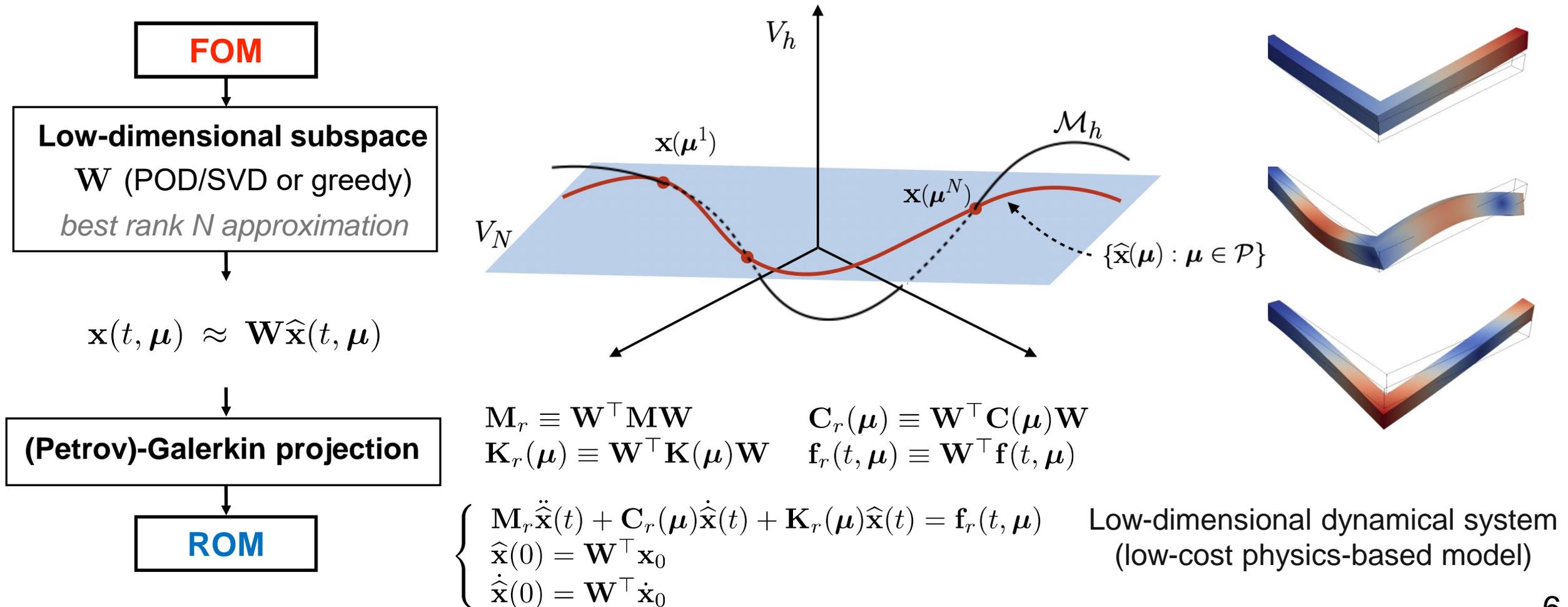
Given initial conditions, boundary conditions, and system parameters compute solution trajectories, **to be compared with sensor recordings**

The need of reduced-order modeling (ROM)

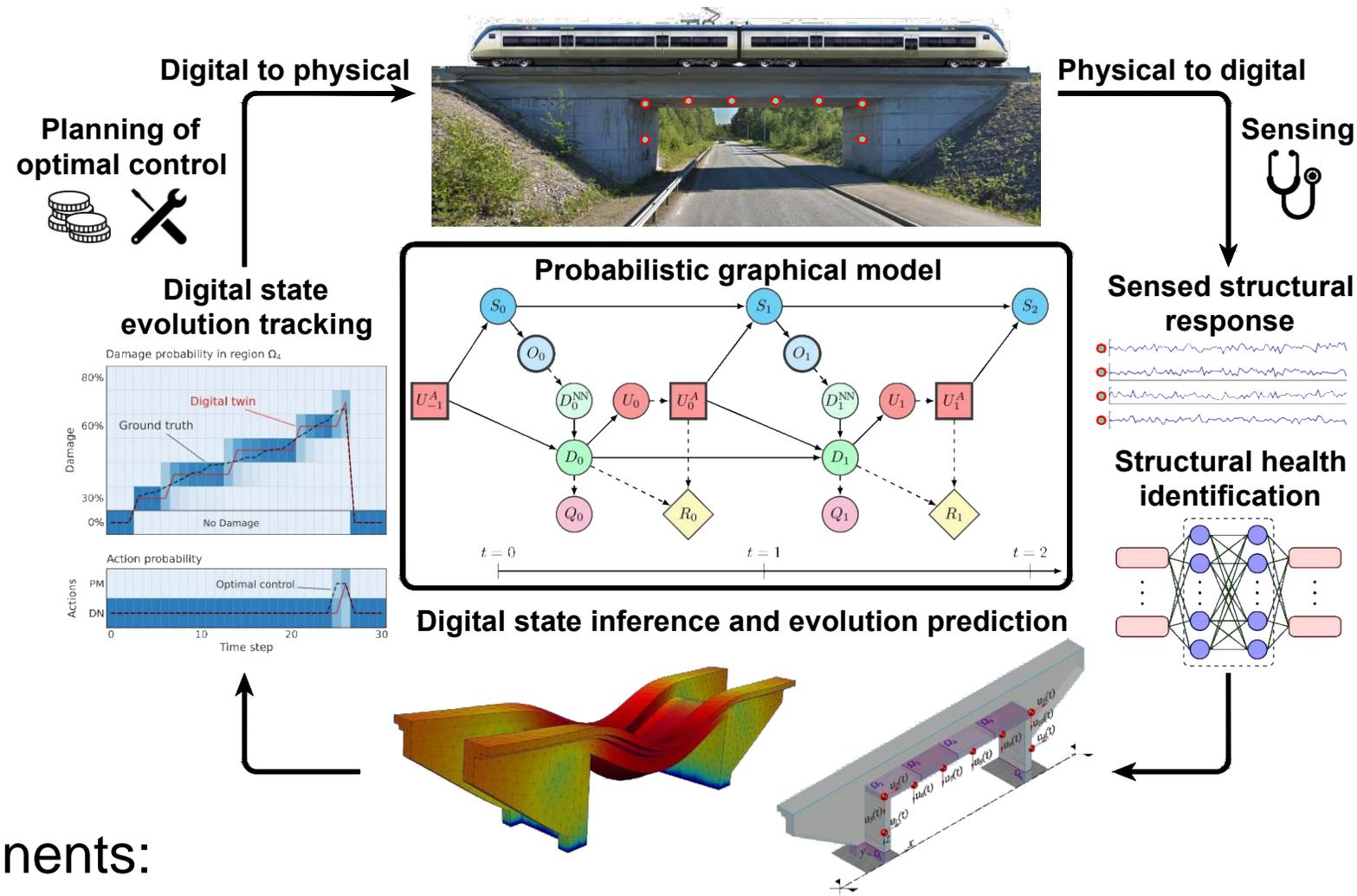
- The offline generation of synthetic training datasets, sufficiently representative of potential damage and operational conditions, may become prohibitive.
- We employ the reduced basis method for parametrized systems (not a restrictive choice).

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Overview: end-to-end information flow



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Data-driven approach to inverse problems – neural network case

\mathcal{F} := Forward operator (parameters \rightarrow measurements)

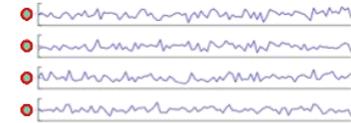
\mathcal{I} := Inverse problem (measurements \rightarrow sought parameters)

\mathcal{I}_{θ^*} := Neural network approximation to \mathcal{I}

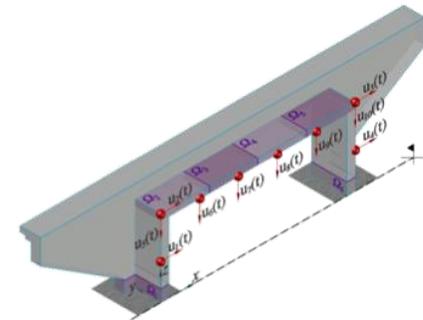
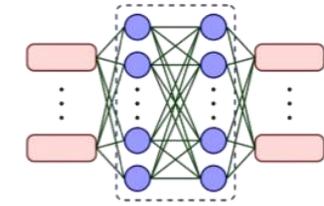
Loss function prototype

$$= \arg \min_{\theta \in \Theta} \sum_j \|(\mathcal{I}_{\theta} \circ \mathcal{F})(\mu_j) - \mu_j\|$$

Sensed structural response



Structural health identification



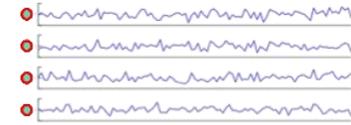
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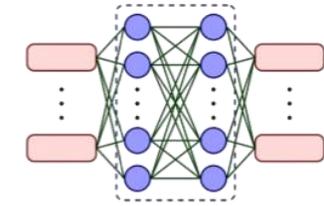
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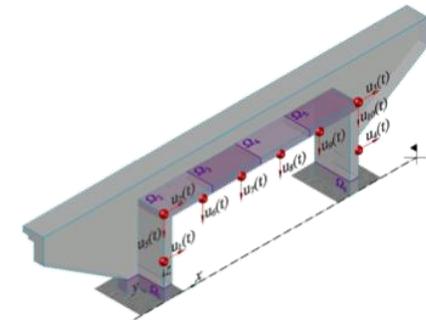
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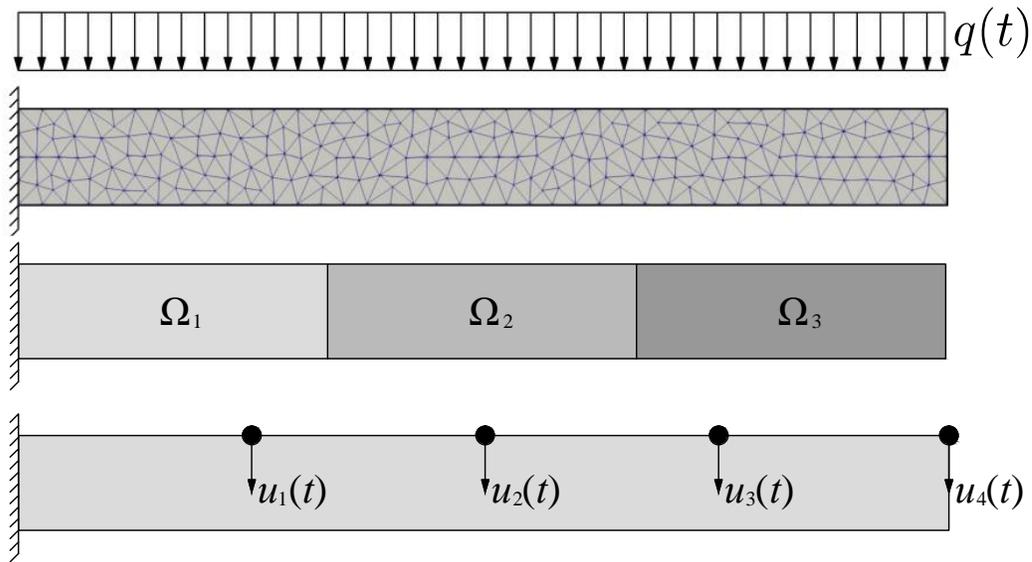
presence, location, severity of damage



The case of SHM:

- ❖ parameters $\boldsymbol{\mu}$: define an expressive representation of the structural health
 - ❖ measurements: experimental (sensors) vs simulated (reduced-order model)
 - ❖ forward operator \mathcal{F} : real vs simulated structural response
- stiffness reduction
 - loose knot bolts
 - crack pattern
 - delamination size

Simulation-based damage detection/localization & quantification

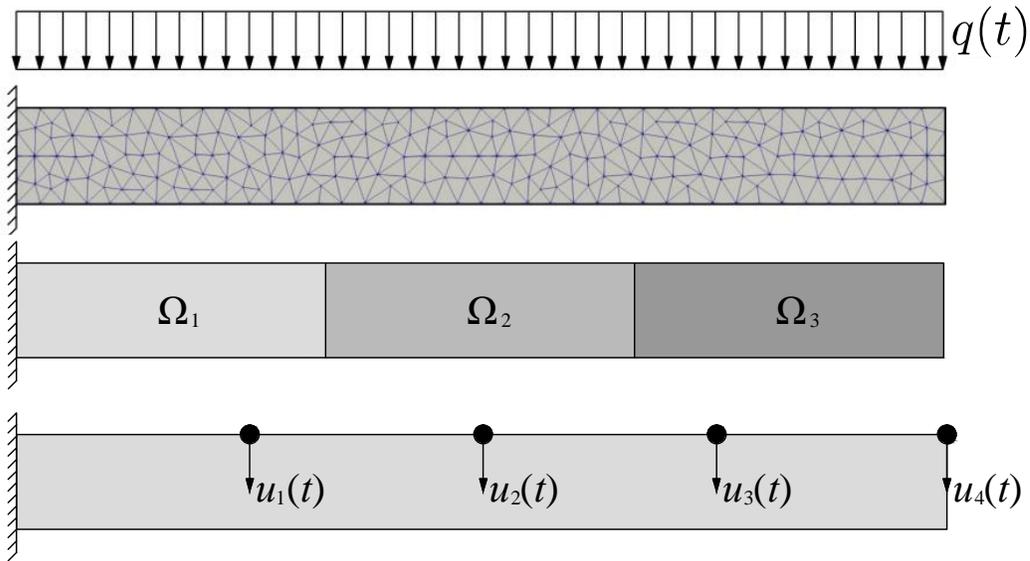


Simulation-based SHM: the problem is traced back to train machine learning models on simulated data.

Damage: introduce damageable regions distributed over the structure and model the effect of damage.

Processed data: vibration recordings shaped as multivariate time series, mimicking a sensor network.

Simulation-based damage detection/localization & quantification



Simulation-based SHM: the problem is traced back to train machine learning models on simulated data.

Damage: introduce damageable regions distributed over the structure and model the effect of damage.

Processed data: vibration recordings shaped as multivariate time series, mimicking a sensor network.

Evaluate forward models to generate training data and train inverse models (offline):

Damage detection/localization as a classification task:

$$\mathcal{D}_{\text{CL}} = \{(\mathbf{U}_i, \mathbf{b}_i)\}_{i=1}^I$$

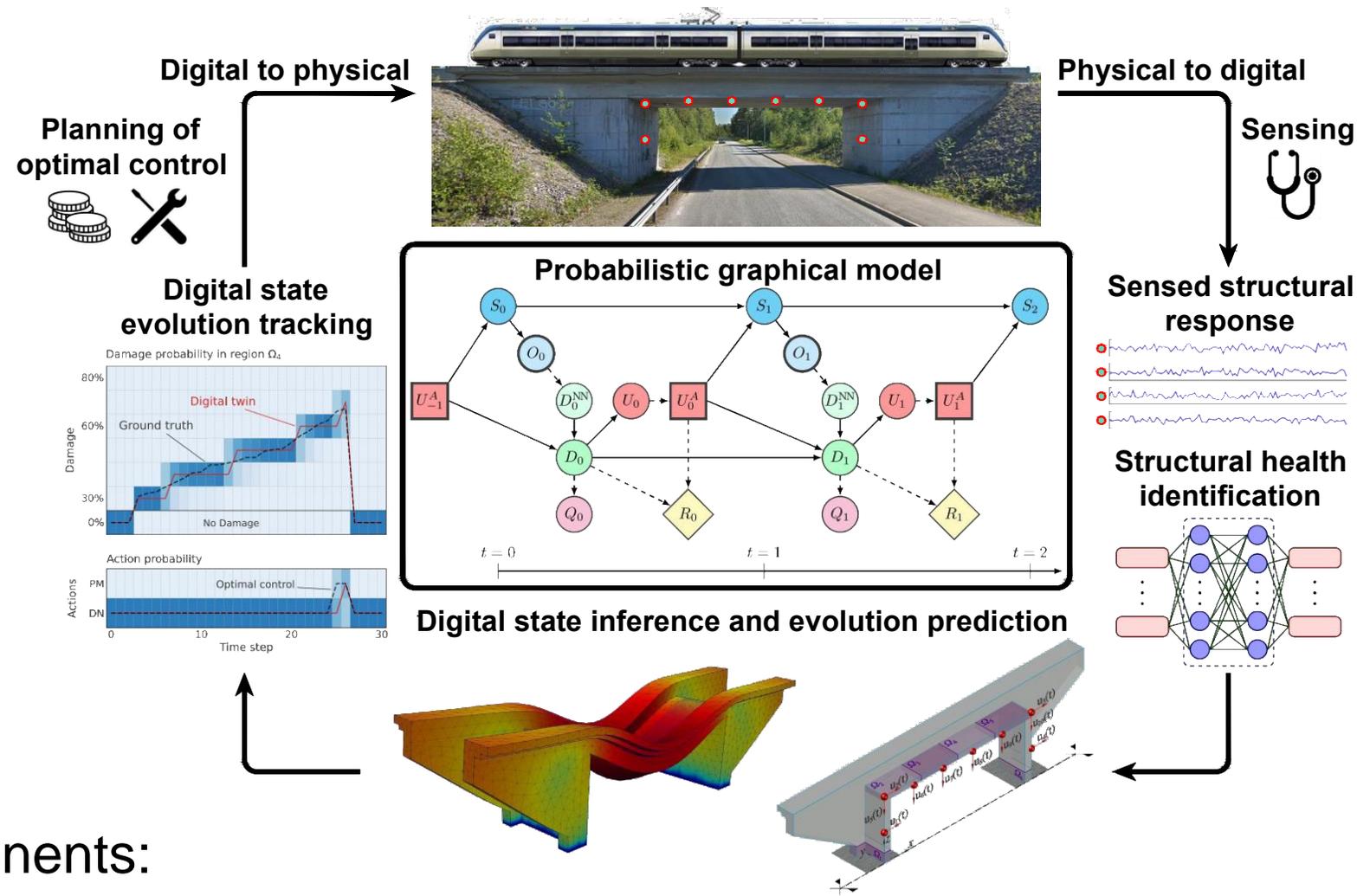
$$\mathcal{L}_{\text{CL}}(\Theta_{\text{CL}}, \mathcal{D}_{\text{CL}}) = -\frac{1}{I} \sum_{i=1}^I \sum_{m=0}^{N_y} b_i^m \log(\hat{b}_i^m)$$

Damage quantification as a regression task:

$$\mathcal{D}_{\text{RG}} = \{(\mathbf{U}_{i_{\text{RG}}}, \delta_{i_{\text{RG}}})\}_{i_{\text{RG}}=1}^{I_{\text{RG}}}$$

$$\mathcal{L}_{\text{RG}}(\Theta_{\text{RG}}, \mathcal{D}_{\text{RG}}) = \frac{1}{I_{\text{RG}}} \sum_{i_{\text{RG}}=1}^{I_{\text{RG}}} (\delta_{i_{\text{RG}}} - \hat{\delta}_{i_{\text{RG}}})^2$$

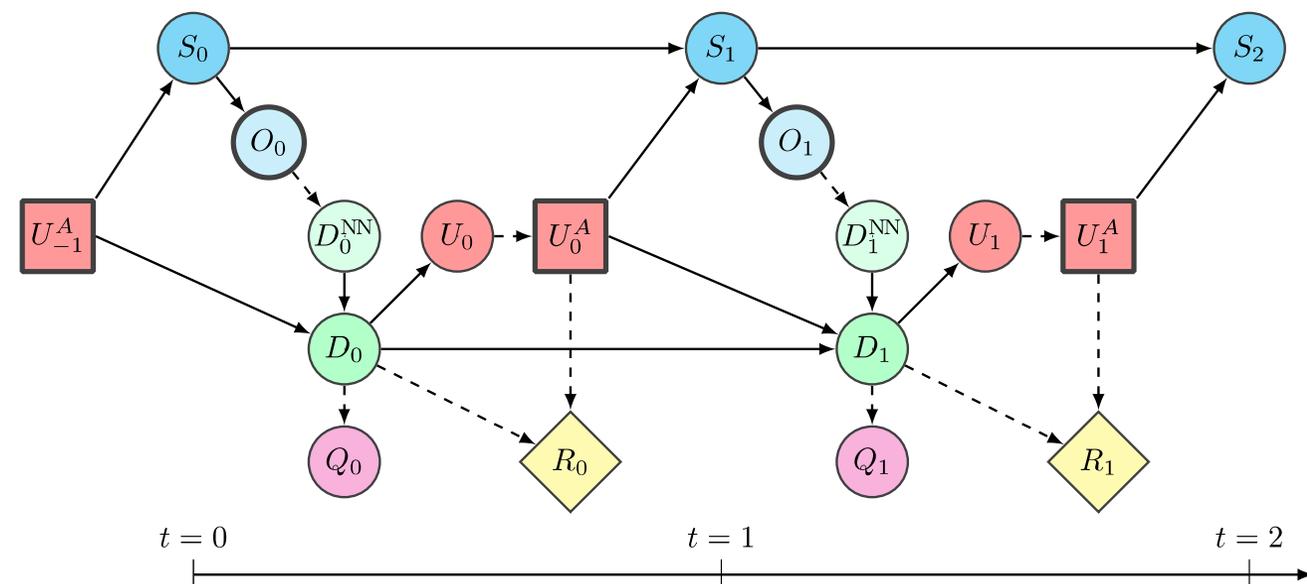
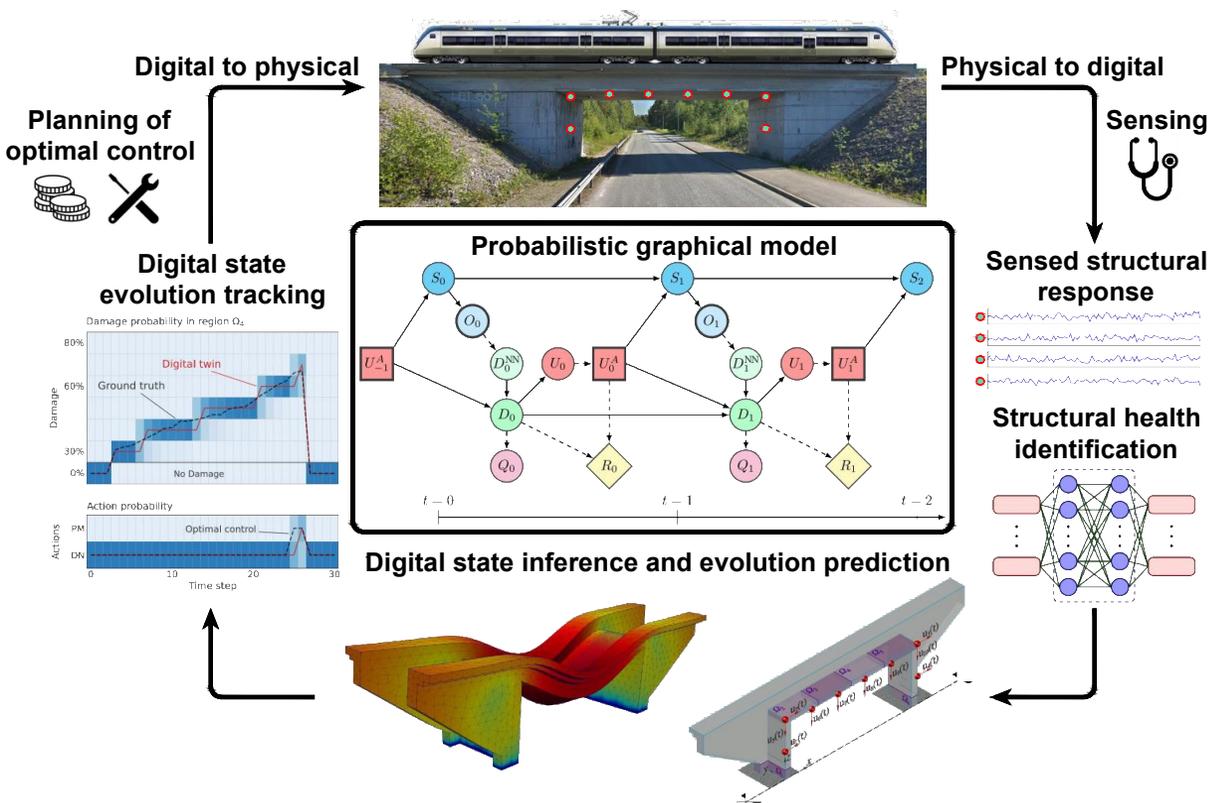
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Probabilistic graphical model encoding the asset-twin system

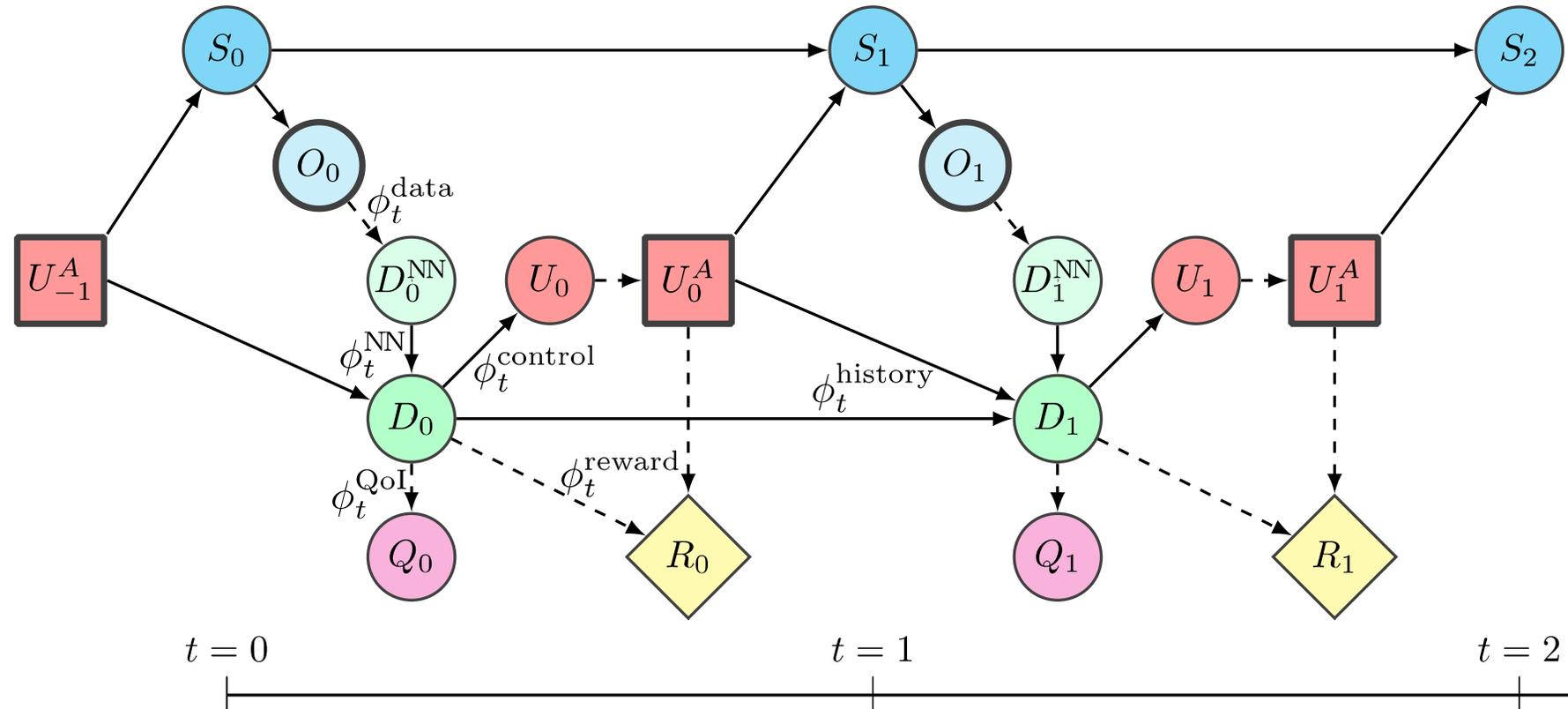


- Physical state: $S_t \sim p(s_t)$ - variability of the asset
- Digital state: $D_t \sim p(d_t)$ - capture the asset variability
- Observations: $O_t \sim p(o_t)$ - from physical to digital flow
- QoI: $Q_t \sim p(q_t)$ - estimated via model output
- Control inputs: $U_t \sim p(u_t)$ - from digital to physical flow
- Reward: $R_t \sim p(r_t)$ - asset-twin performance

Key assumptions:

- Physical state only observable indirectly via the sensed structural response.
- Markovianity of physical and digital states.

Belief state factorization



Physical state: $S_t \sim p(s_t)$
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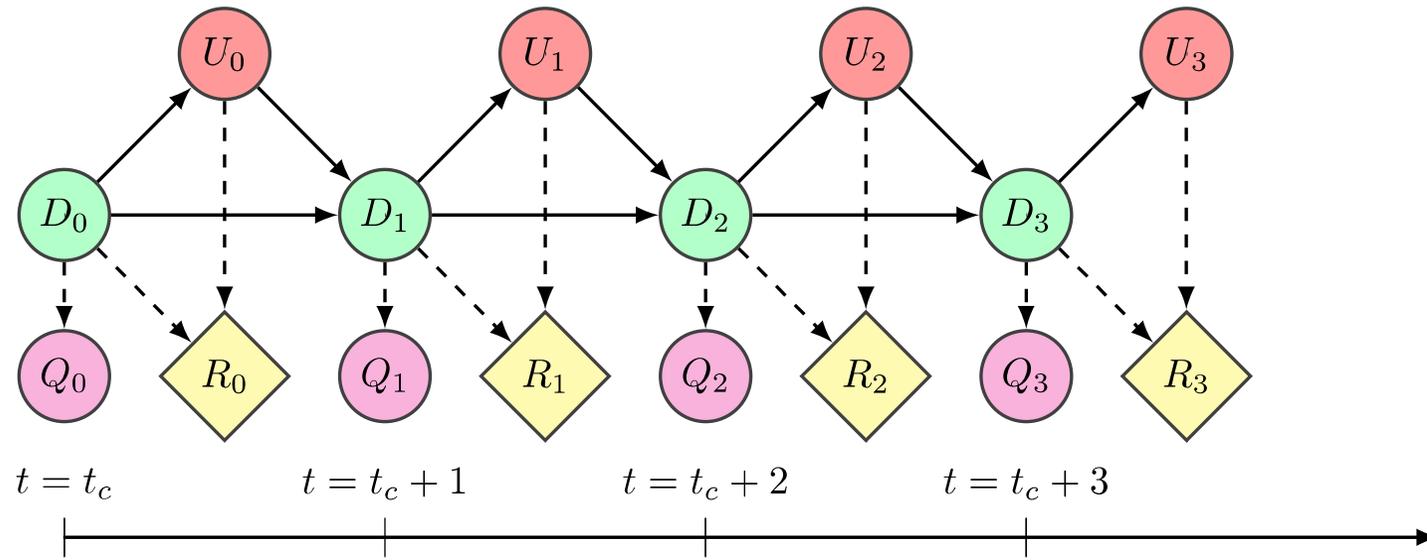
$$p(D_0^{\text{NN}}, \dots, D_{t_c}^{\text{NN}}, D_0, \dots, D_{t_c}, Q_0, \dots, Q_{t_c}, R_0, \dots, R_{t_c}, U_0, \dots, U_{t_c} | o_0, \dots, o_{t_c}, u_0^A, \dots, u_{t_c}^A)$$

$$\propto \prod_{t=0}^{t_c} \left[\phi_t^{\text{data}} \phi_t^{\text{history}} \phi_t^{\text{NN}} \phi_t^{\text{QoI}} \phi_t^{\text{control}} \phi_t^{\text{reward}} \right],$$

$$\phi_t^{\text{data}} = p(O_t = o_t | D_t^{\text{NN}}), \quad \phi_t^{\text{history}} = p(D_t | D_{t-1}, U_{t-1}^A = u_{t-1}^A), \quad \phi_t^{\text{NN}} = p(D_t | D_t^{\text{NN}}),$$

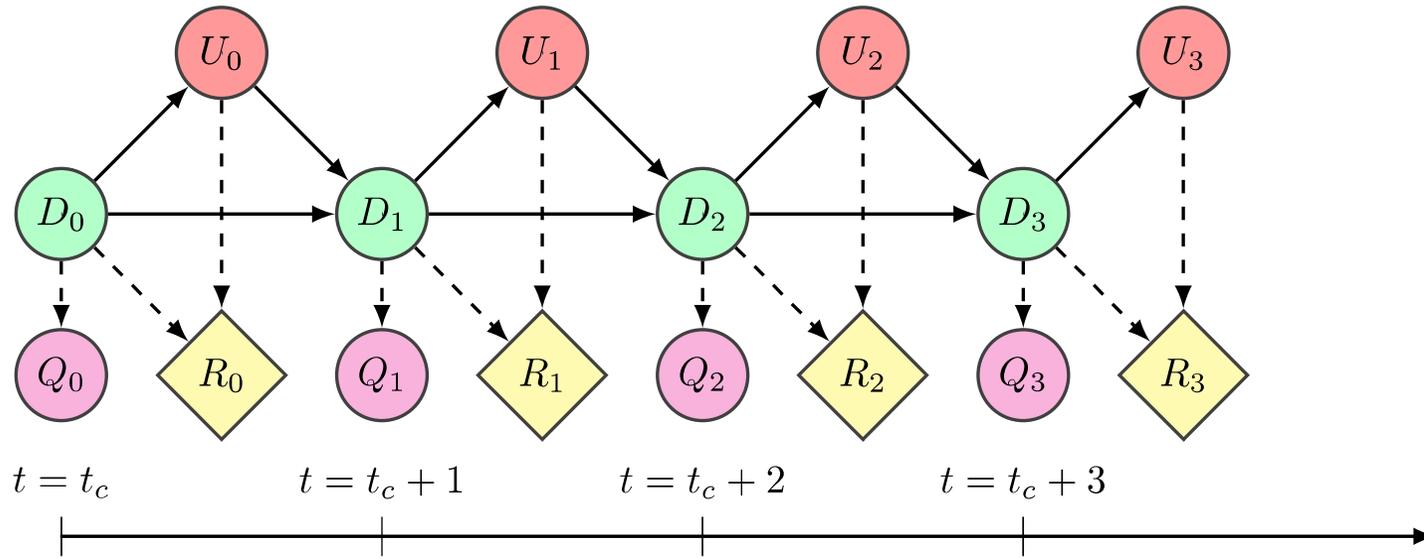
$$\phi_t^{\text{QoI}} = p(Q_t | D_t), \quad \phi_t^{\text{reward}} = p(R_t | D_t, U_t^A = u_t^A), \quad \phi_t^{\text{control}} = p(U_t | D_t).$$

Planning of optimal control & extension to prediction



- **Forecasting/maintenance planning** from the updated digital state at the current time step (no data assimilation).
- Unroll the portion of the graph relative to digital state, control inputs, reward and quantities of interest.

Planning of optimal control & extension to prediction



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Planning of optimal control

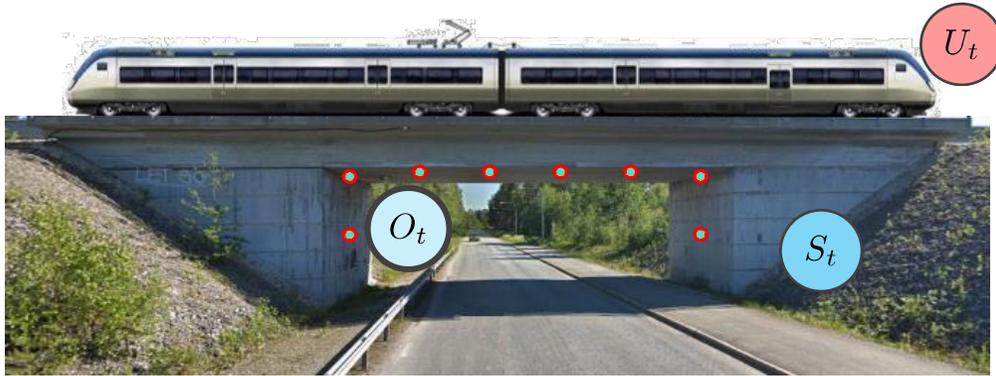
$$\pi(D_t) = \arg \max_{\pi} \sum_{t=0}^{+\infty} \gamma^t \mathbb{E}[R_t]$$

$\phi_t^{\text{control}} = p(U_t | D_t)$ Control policy maps the digital state belief onto actions

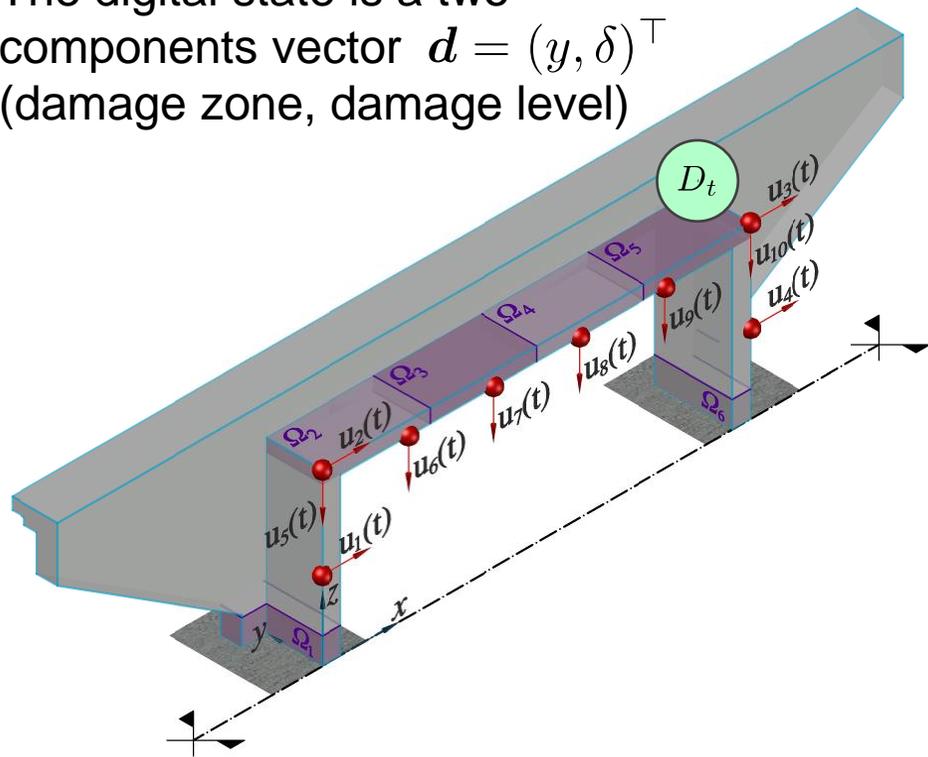
Multi-objective planning reward function
 $R_t(U_t, D_t) = R_t^{\text{control}}(U_t) + \alpha R_t^{\text{health}}(D_t)$

$$p(D_0^{\text{NN}}, \dots, D_{t_c}^{\text{NN}}, D_0, \dots, D_{t_p}, Q_0, \dots, Q_{t_p}, R_0, \dots, R_{t_p}, U_0, \dots, U_{t_p} | o_0, \dots, o_{t_c}, u_0^A, \dots, u_{t_c}^A) \\ \propto \prod_{t=0}^{t_p} \left[\phi_t^{\text{history}} \phi_t^{\text{QoI}} \phi_t^{\text{control}} \phi_t^{\text{reward}} \right] \prod_{t=0}^{t_c} \left[\phi_t^{\text{data}} \phi_t^{\text{NN}} \right]$$

Hörnefors railway bridge

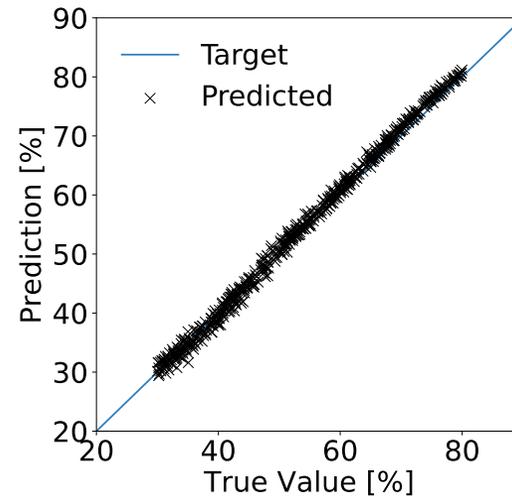
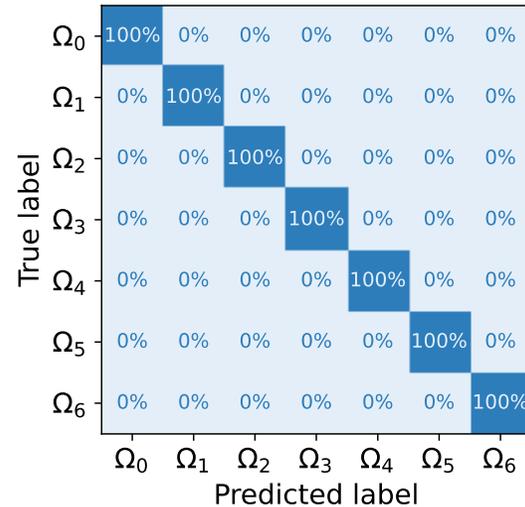


The digital state is a two components vector $\mathbf{d} = (y, \delta)^\top$ (damage zone, damage level)



Data assimilation:

$$\phi_t^{\text{data}} = p(O_t = o_t | D_t^{\text{NN}})$$

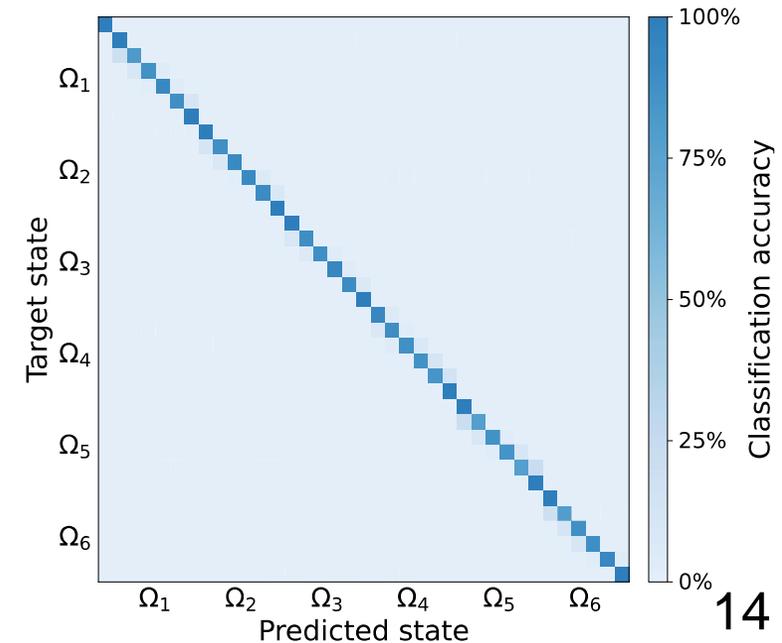


Damage modeling:

Undamaged case
+ 6 damageable zones

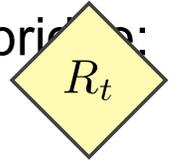
Stiffness reduction in the range (30%,80%), 6 intervals discretiz. (37 possible structural states)

$$\phi_t^{\text{NN}} = p(D_t | D_t^{\text{NN}})$$



Possible control inputs

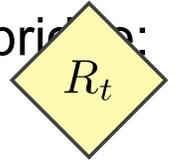
- “**Do nothing**” (DN): the physical state evolves according to a stochastic deterioration process.
- “**Perfect maintenance**” (PM): A maintenance action is performed and the asset returns from its current condition to the damage-free state.
- “**Restrict operational conditions**” (RE): only light weight trains are allowed to cross the bridge; lower deterioration rate, but also lower revenue generated by the infrastructure.



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Transition models

Each control input is provided with a conditional probability table describing the corresponding transition model.

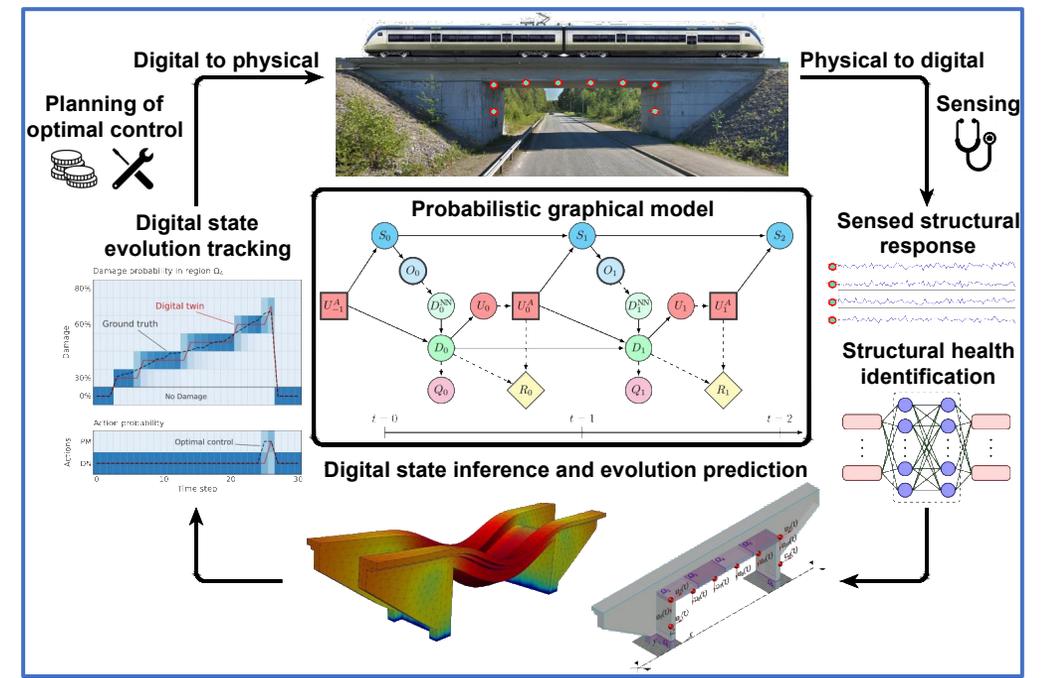
$$\phi_t^{\text{history}} = p(D_t | D_{t-1}, U_{t-1}^A = u_{t-1}^A)$$

- **DN**: damage may start in any subdomain with 0.1 probability, and then grow to the next δ interval with the same probability.
- **PM**: the belief about the digital state is mapped to the undamaged condition, independently of the current condition.
- **RE**: damage may start in any subdomain, with 0.03 probability, and then grow to the next δ interval with the same probability.

Ground-truth evolution model

To run a digital twin simulation we prescribe a (simulated) stochastic degradation process: the digital twin is dynamically updated and used to drive maintenance planning.

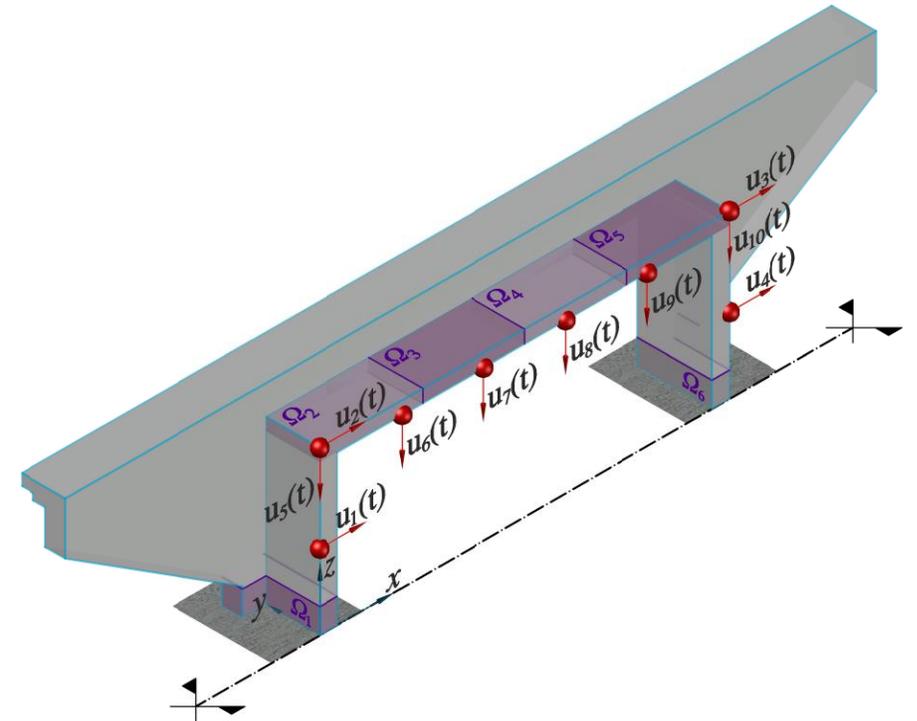
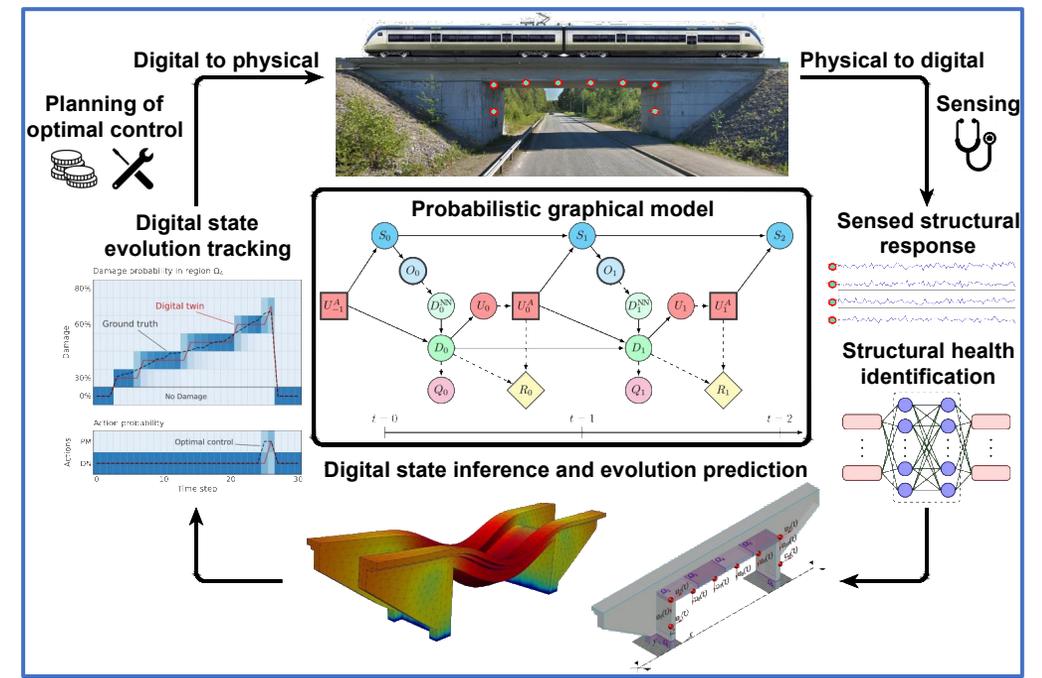
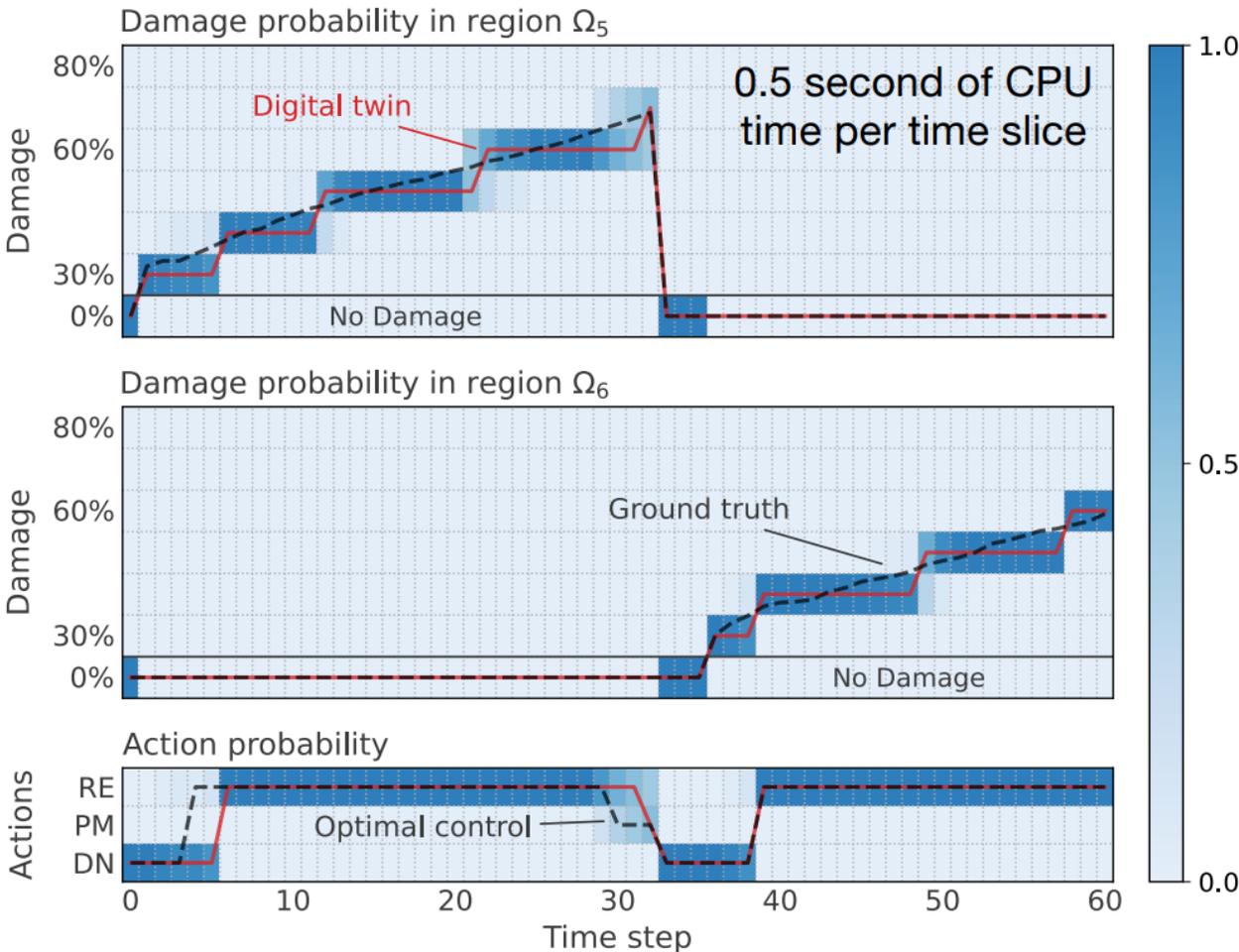
Damage may develop in any of the predefined regions and then propagate with δ increments sampled from a Gaussian pdf, chosen according to the last enacted control input.



Ground-truth evolution model

To run a digital twin simulation we prescribe a (simulated) stochastic degradation process: the digital twin is dynamically updated and used to drive maintenance planning.

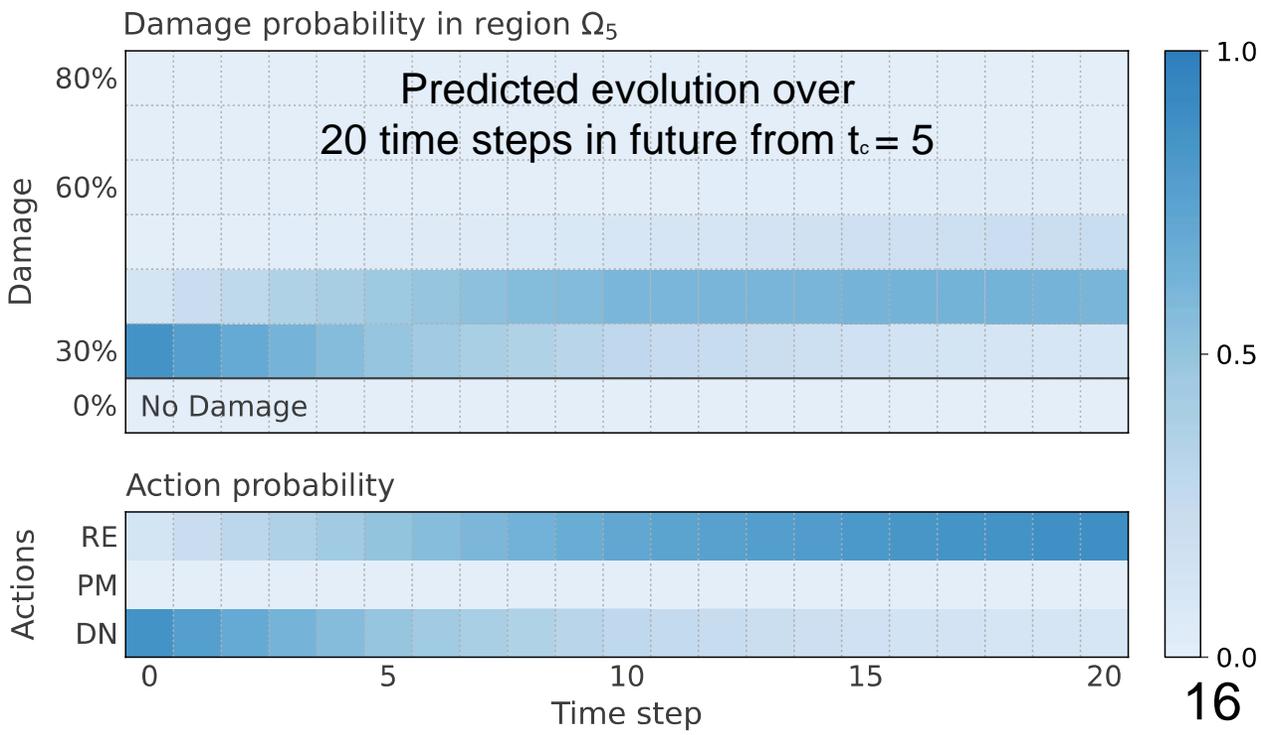
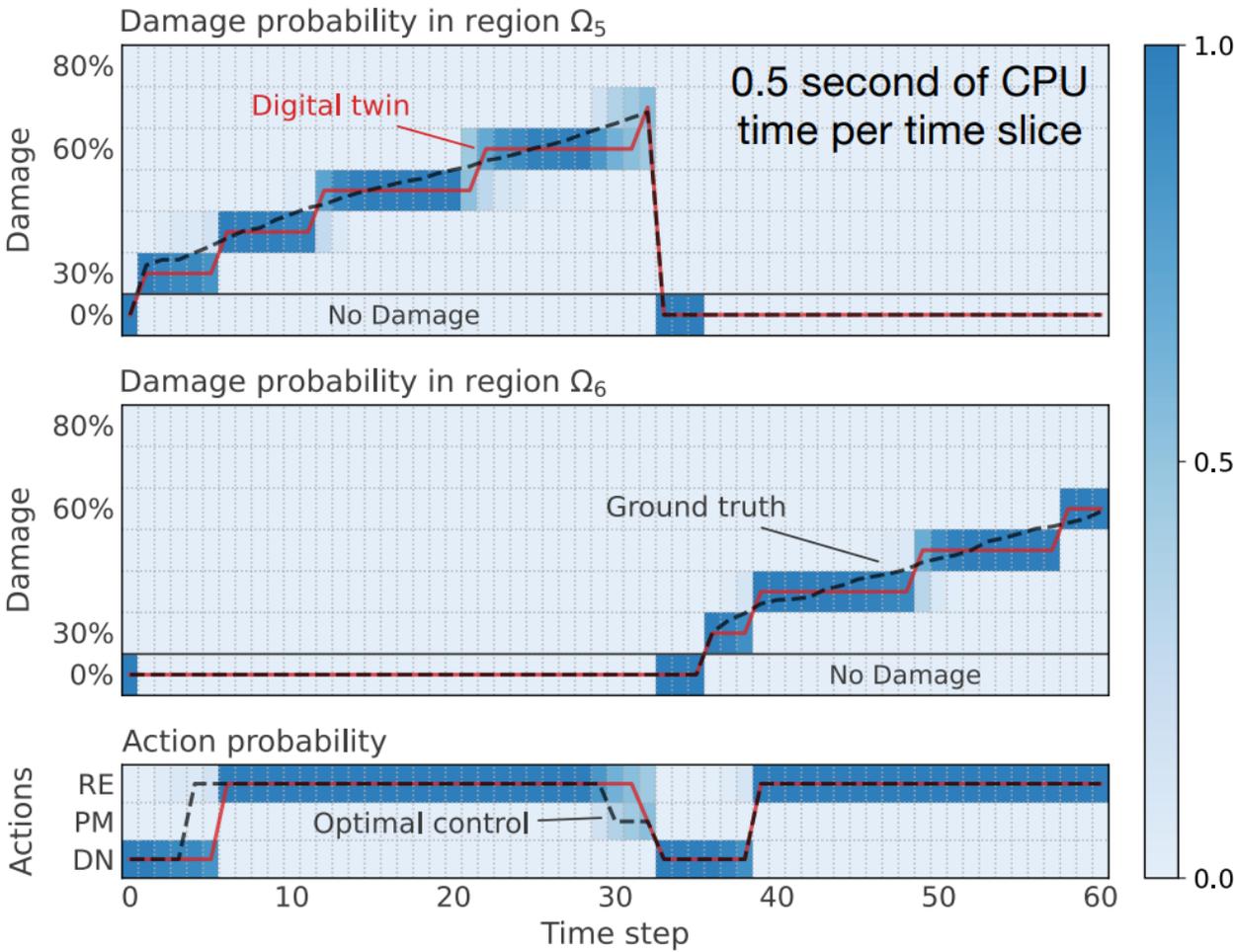
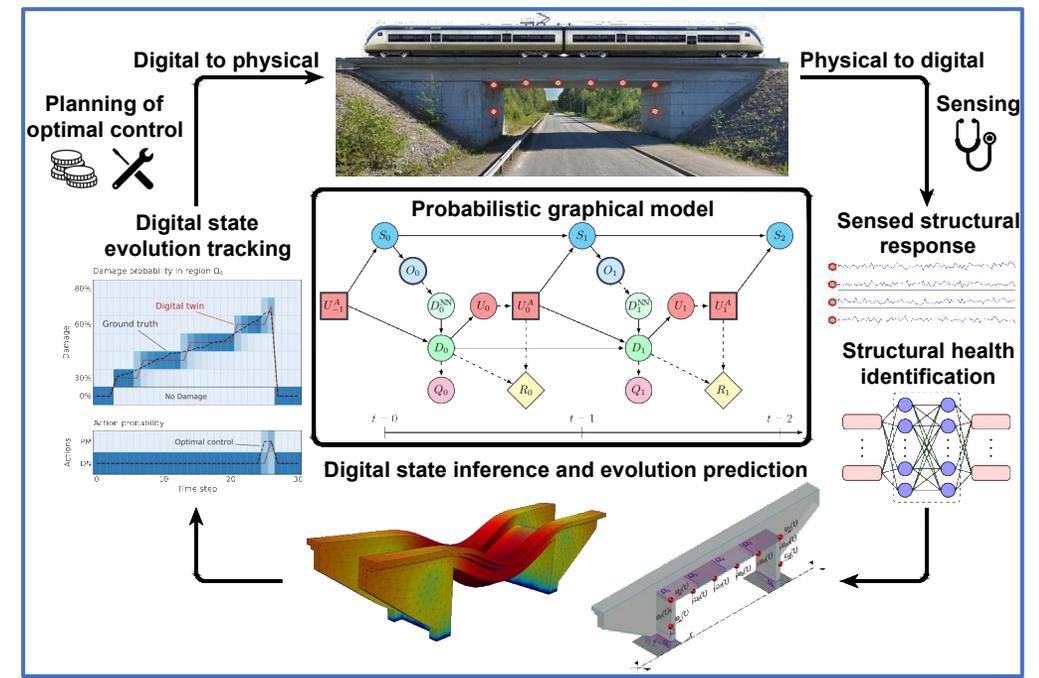
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Ground-truth evolution model

To run a digital twin simulation we prescribe a (simulated) stochastic degradation process: the digital twin is dynamically updated and used to drive maintenance planning.

Damage may develop in any of the predefined regions and then propagate with δ increments sampled from a Gaussian pdf, chosen according to the last enacted control input.



Future developments

- The transition models are currently prescribed by the user. To better characterize them, it would be useful to **update the transition dynamic models from the online data stream**. This would result in a more calibrated prediction of the digital state expected evolution.
- The planning problem is currently solved by considering an infinite planning horizon, not realistic for civil structures. A more viable alternative would be **a finite planning horizon representing the design lifetime of the asset and, e.g., reinforcement learning**.
- **Quantities of interest** such as modal quantities or full response fields obtained through ROMs, currently not exploited, could be used **to perform posterior predictive checks on the tracking capabilities of the digital twin**, useful to evaluate how well it matches the reality.

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THANK YOU!



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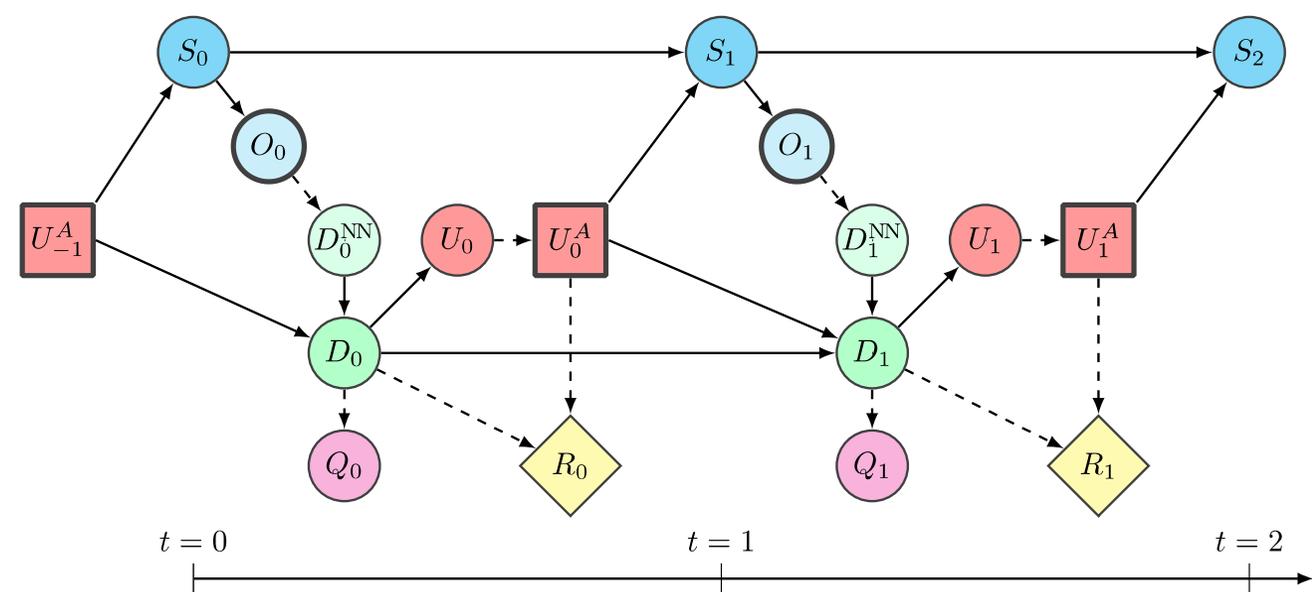
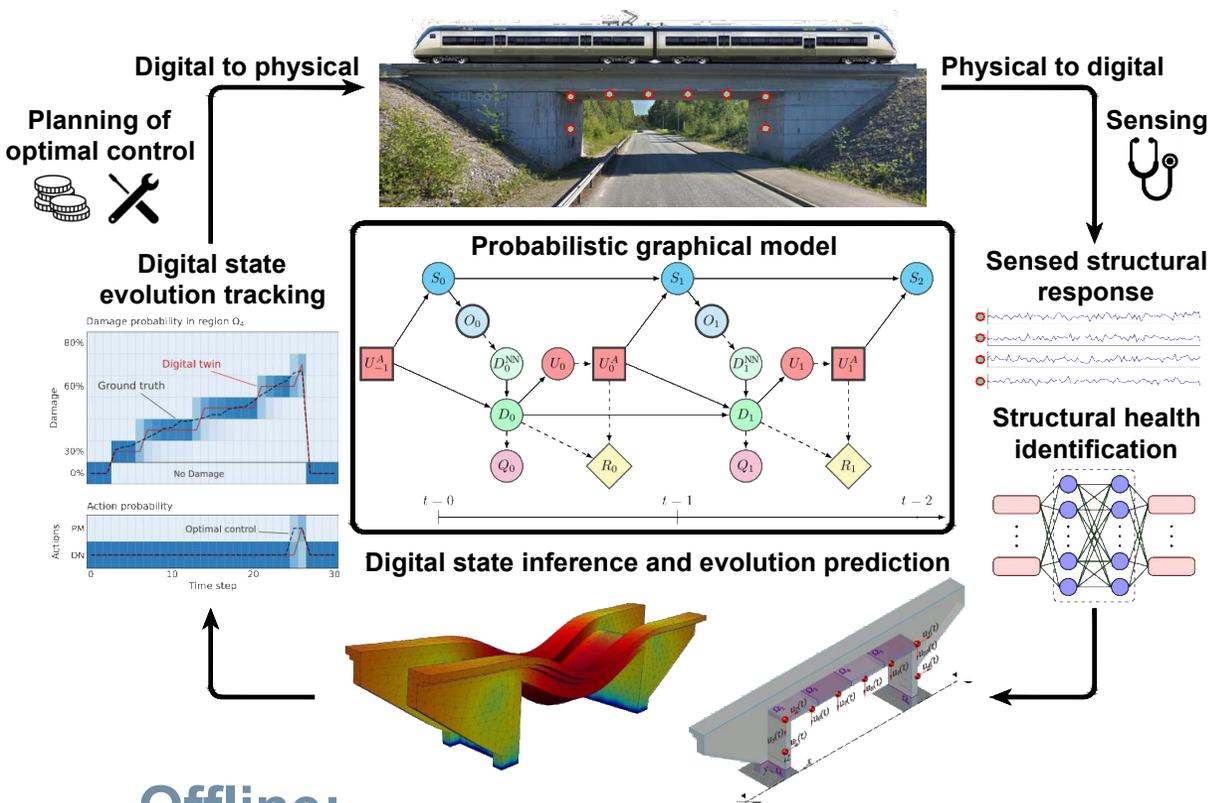


TEXAS

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Probabilistic graphical model encoding the asset-twin system



Offline:

- Derive the reduced-order model
- Populate the training dataset
- Train the SHM deep learning models
- Estimate the transition models ϕ_t^{history} from historical data of similar structures
- Compute the control policy (planning)

Online (repeats indefinitely):

- Assimilate incoming observational data
- Inference of digital state and control inputs
- Update ϕ_t^{history} on the online data stream
- Compute quantities of interest
- Predict the digital state evolution
- Enact the suggested control action

Bonus