On Some Optimization Problems in Power Energy Industry

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Joint work with
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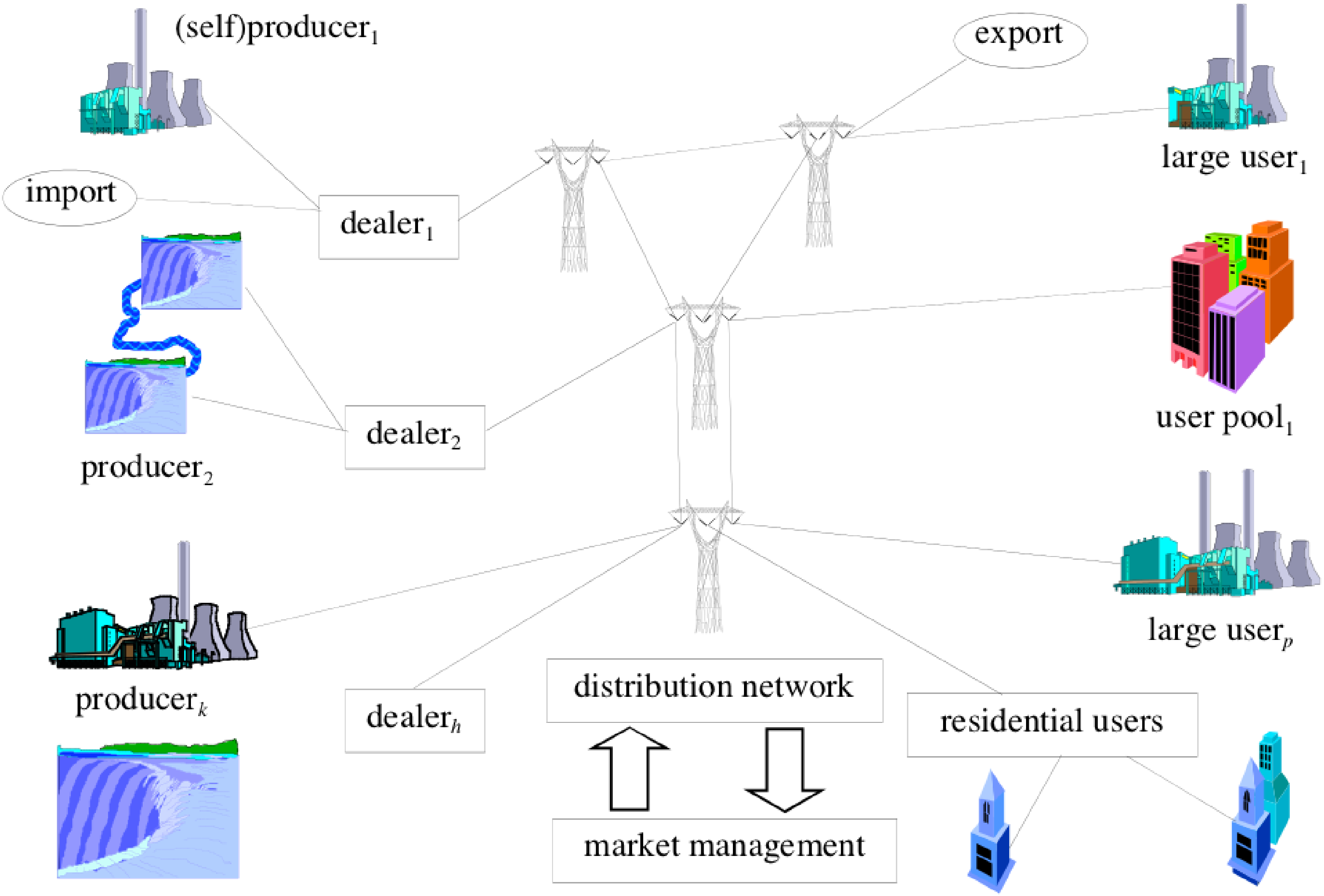


(self)producer$_1$

export

import

dealer$_1$

large user$_1$

user pool$_1$

dealer$_2$

producer$_2$

large user$_p$

producer$_k$

dealer$_h$

distribution network

residential users

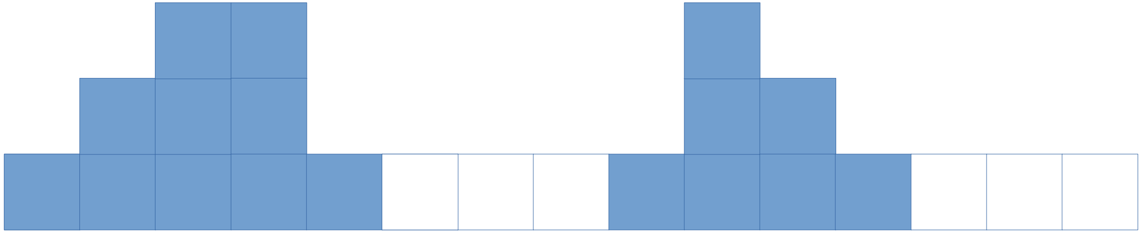
market management

Optimization problems for Power Energy

1. Production: localization and design of production plants, daily tactical optimization, strategic bidding;
2. Distribution: grid safety and grid design; tactical and online grid management, recovery management;
3. Market: price clearing methodologies, strategic analysis of the market, reserve handling and ancillary services;
4. New dealers: self producers, energy communities, renewable sources, traders.

Production Problem: Unit Commitment

1. Scheduling generation units over a short time horizon.
2. Decide which generations are active at each time instant → binary variables
3. Decide the amount of energy generated by each unit → continuous variables



Production Problem: Unit Commitment (2)

1. Constraint 1: minimum and maximum produced power.
2. Constraint 2: each unit must remain in the same status (on/off) for a minimum time.
3. Constraint 3: the power can increase/decrease of a maximum amount.
4. Objective Function: non linear on the produced power and on the status.
5. Satisfying some global constraints: demand constraints, grid constraints, reserve constraints.

Two main approaches

Lagrangian Relaxation
and Decomposition

Mixed-Integer (Non) Linear
Programming Formulations
MI(N)LP +off-of-shelf MIP solvers

Some Results (1)

Lagrangian Relaxation
and Decomposition

Decomposes into single-unit subproblems (1UC)

Dynamic Programming
algorithm for 1UC

Including ramp-constraints
and nonlinear costs
$O(n^3)$ complexity

Some results (2)

Major problems:

1. Non linear objective function
2. History Start-up costs
3. Poor formulations

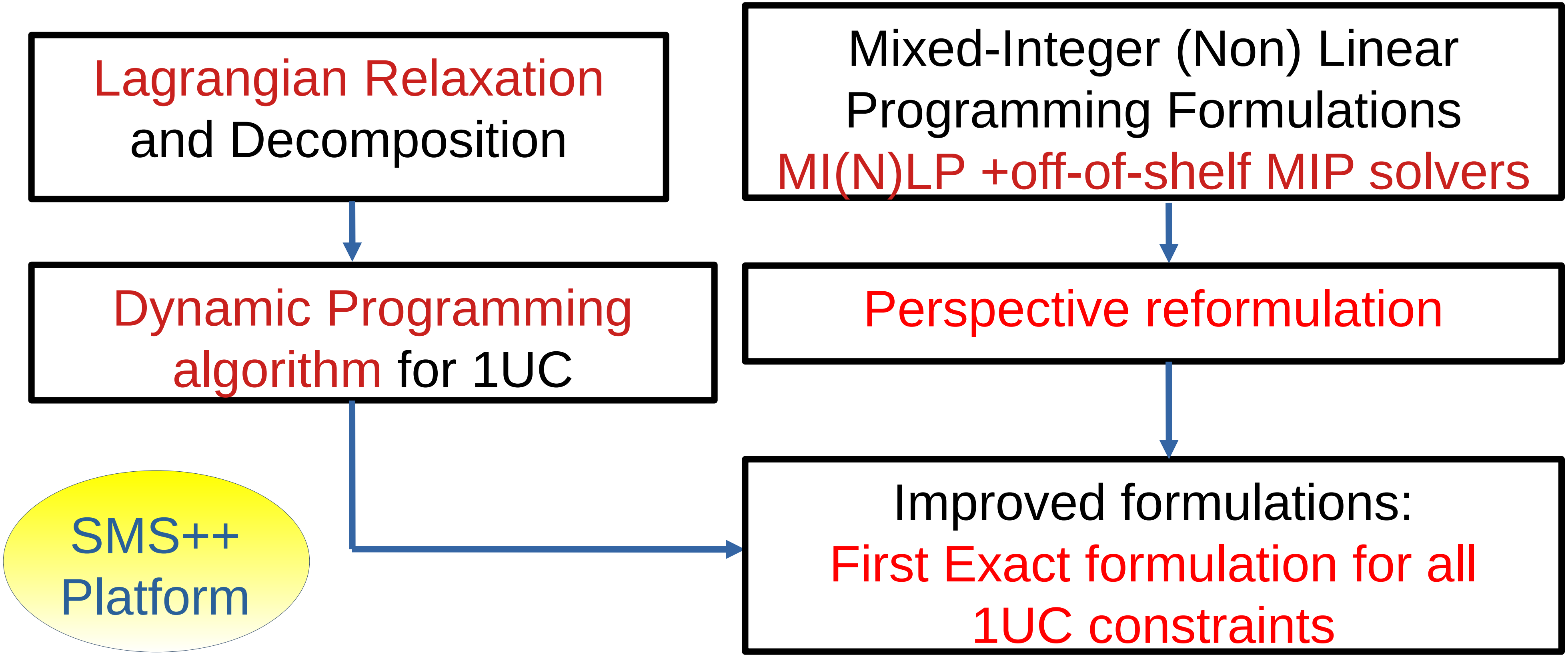
Multiple applications

1. Portfolio Optimization
2. Cardinality constraints
3. Machine Learning

Mixed-Integer (Non) Linear Programming Formulations
MI(N)LP +off-of-shelf MIP solvers

Perspective reformulation:
Perspective Cuts
Projected reformulation

Some results (3)



Lagrangian Relaxation and Decomposition

Dynamic Programming algorithm for 1UC

SMS++ Platform

Mixed-Integer (Non) Linear Programming Formulations

MI(N)LP +off-of-shelf MIP solvers

Perspective reformulation

Improved formulations:

First Exact formulation for all 1UC constraints

Optimal Power Flow (OPF)

1. Power must be distributed from production sites to consumers
2. **Kirchoff and Ohm’s laws** should be respected
3. Different models: **AC vs DC**
    - OPF in DC → easy LP models
    - **OPF in AC** → difficult NLP models in complex variables
4. Braess paradox may occur: adding branches may increase the total cost
    - Decide which branches should be open/closed at each period
    - **Optimal Transmission Switching** (OTS) difficult → MILP models even in DC

Optimal Transmission Switching (OTS)

1. DC approximation: angle-based formulation vs cycle-based formulation
2. Angle-based formulation: at each node a voltage-angle should be assigned
3. current flow on a branch depend on the difference of voltage between nodes
4. Cycle-based formulation: sum of voltages on each cycle is zero (KVL)
    1. Cycles are exponentially many!!!
    2. Complexity of separation of cycle inequalities for OTS: unknown

$\Rightarrow$New result: an exact separation algorithm for OTS cycle inequalities

$\Rightarrow$Complexity: pseudopolynomial!

Capacity allocation problem

Node/branch failures or sudden consumption changes may cause cascade black-outs!

Static problem
design a resilient grid, where no single branch failure can cause a cascade
Dynamic problem
design a grid where cascades can start, but are of limited consequences

Results
1. a DC approximation model
2. valid inequalities and related separation procedures
3. exact and heuristic algorithms

Generation &Transmission Expansion Planning (GTEP)

1. Decide capacities and locations (of both generators and transmission branches) that are optimal for the electric power system in the long-term
    - Minimize the sum of the investment and operational costs
    - Meet peak demand with reserve margins
    - Achieve predefined policy goals
    - Account for technical constraints
    - Account for long-term uncertainty in fuel and CO2 prices

GTEP with cost recovery

A new MINLP model

1. Revenue Adequacy (RA) constraint: each generator is guaranteed to recover its investment and operational costs through energy payments
2. Quantities and prices are simultaneously determined by LP duality theory
3. Bilinear terms in both the RA constraint and in the objective function
    - McCormick relaxation and discretization technique

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Thank you for your attention.